Neutron Interferometry

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What is an interferometer?

An interferometer is a device that measures the relative phase between coherent waves by detecting the interference pattern.
Michelson Interferometer

Mach-Zender Interferometer
Perfect Crystal LLL Neutron Interferometer

Bragg condition: $n\lambda = 2d \sin \theta$

$d = \text{lattice spacing}$
Perfect Crystal LLL Neutron Interferometer
Perfect Crystal LLL Neutron Interferometer
Perfect Crystal LLL Neutron Interferometer
Nuclear Phase Shift
Nuclear Phase Shift

index of refraction: \[ n = 1 - \frac{N b \lambda^2}{2\pi} \]

relative phase shift:

\[ \Delta \chi = k_0 \ell - nk_0 \ell = N b \lambda \frac{D}{\cos \theta} \]
Interferogram

Diagram showing a neutron beam, phase flag, H beam, \(^3\)He detectors, and O beam.
Interferogram

O beam: \[ I_o = A\left[1 + f \cos(\chi_2 - \chi_1)\right] \]

H beam: \[ I_H = B - Af \cos(\chi_2 - \chi_1) \]

contrast \[ f = \frac{C_{\text{max}} - C_{\text{min}}}{C_{\text{max}} + C_{\text{min}}} \quad (\text{O-beam}) \]
Precision Phase Shift Measurement

\[ \Delta \chi = N \hbar \lambda \frac{D}{\cos \theta} \]

Example:
aluminum sample, \( \lambda = 2.70 \text{ Å} \), \( \langle 111 \rangle \) reflection

\[ D = 100 \mu \text{m} \Rightarrow \Delta \chi = 2\pi \]
Non-Dispersive Geometry

\[ \Delta \chi = 2NbdD \]

path length \( \ell = \frac{D}{\sin \theta} \)

independent of \( \lambda \)
Perfect Crystal LLL Neutron Interferometer
Precise neutron-interferometric measurement of the coherent neutron-scattering length in silicon

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The neutron-interferometry (NI) technique provides a precise and direct way to measure the bound, coherent scattering lengths $b$ of low-energy neutrons in solids, liquids, or gases. The potential accuracy of NI to measure $b$ has not been fully realized in past experiments, due to systematic sources of error. We have used a method which eliminates two of the main sources of error to measure the scattering length of silicon with a relative standard uncertainty of 0.005%. The resulting value, $b$ = 4.1507(2) fm, is in agreement with the current accepted value, but has an uncertainty five times smaller. [S1050-2947(98)04808-2]

PACS number(s): 03.75.Dg, 07.60.Ly, 61.12.-q
net phase shift: \( \Theta(\varepsilon_0, \gamma_0) = 248\pi + 0.455(7) \) radians

\[ b_{\text{coh}} = 4.15041(21) \text{ fm} \]
Skew-Symmetric Neutron Interferometer
NIST perfect crystal silicon interferometers
Components:
1. Collimator/shutter
2. Helium filled beam transport tube
3. Focusing pyrolytic graphite monochromator
4. Outer environmental enclosure
5. Primary vibration isolation stage
6. Acoustic and thermal isolation enclosure
7. Secondary vibration isolation stage
8. Enclosure for interferometer and detectors
S18 Neutron Interferometer at the Institut Laue-Langevin
"Geometric" Bragg Reflection

path 2 travels addition distance $AB + BC = 2d \sin \theta$

relative phase shift: $\Delta \phi = \frac{2d \sin \theta}{\lambda} \times 2\pi$

condition for constructive interference: $n\lambda = 2d \sin \theta$ (Bragg's Law)
Kinematic Bragg Diffraction
Kinematic Bragg Diffraction
Kinematic Bragg Diffraction
Kinematic Bragg Diffraction
Dynamical Diffraction Theory

\[ \vec{k}_0 = \text{inter} \quad \text{for internal scattered wave} \]

\[ \vec{k}_H = \text{ext} \quad \text{for external scattered wave} \]

\[ \vec{H} = \text{Bragg vector} \]

\[ |\vec{H}| = \frac{2\pi n}{d} \]

\[ \vec{K}_0 = \text{internal forward scattered wave} \]

\[ \vec{K}_H = \text{external forward scattered wave} \]

Bragg condition:

\[ \vec{K}_H - \vec{K}_0 = \vec{H} \]

Solve Schrödinger Eqn. inside crystal:

\[ \left( \nabla^2 + k_0^2 \right) \Psi(\vec{r}) = v(\vec{r}) \Psi(\vec{r}) \]

with \( v(\vec{r}) = 4\pi \sum_i b_i \delta (\vec{r} - \vec{r}_i) = \sum_n \nu_{H_n} e^{i\vec{H}_n \cdot \vec{r}} \)
Dynamical Diffraction Theory

Dispersion Equation: \[
\left( K^2 - K_0^2 \right) \left( K^2 - K_H^2 \right) = v_H^2
\]

approximate: \[
\left( K - K_0 \right) \left( K - K_H \right) = \frac{v_H^2}{4k_0^2}
\]

quadratic equation

2 solutions for \( K_0 \)
Dynamical Diffraction Theory

off Bragg
1 solution

on Bragg
4 solutions
index of refraction is double-valued

relative index of refraction

α branch
β branch

misset from Bragg (arcsec)
Dynamical Diffraction Theory

internal wave function: \( \Psi(\vec{r}) = \psi_0^\alpha e^{i\vec{K}_0^\alpha \cdot \vec{r}} + \psi_0^\beta e^{i\vec{K}_0^\beta \cdot \vec{r}} + \psi_H^\alpha e^{i\vec{K}_H^\alpha \cdot \vec{r}} + \psi_H^\beta e^{i\vec{K}_H^\beta \cdot \vec{r}} \)
Dynamical Diffraction Theory

internal wave function: \( \Psi(\vec{r}) = \psi_0^\alpha e^{i\vec{K}_0^\alpha \cdot \vec{r}} + \psi_0^\beta e^{i\vec{K}_0^\beta \cdot \vec{r}} + \psi_H^\alpha e^{i\vec{K}_H^\alpha \cdot \vec{r}} + \psi_H^\beta e^{i\vec{K}_H^\beta \cdot \vec{r}} \)

\[
\begin{align*}
\psi_0^\alpha &= \frac{1}{2} \left[ 1 - \frac{y}{\sqrt{1 + y^2}} \right] A_0 \\
\psi_0^\beta &= \frac{1}{2} \left[ 1 + \frac{y}{\sqrt{1 + y^2}} \right] A_0 \\
\psi_H^\alpha &= -\frac{1}{2} \left[ \frac{1}{\sqrt{1 + y^2}} \right] A_0 \\
\psi_H^\beta &= +\frac{1}{2} \left[ \frac{1}{\sqrt{1 + y^2}} \right] A_0 \\
y &= \frac{k_0 \sin 2\theta_B}{2\nu_H} \delta \theta
\end{align*}
\]

misset parameter
Dynamical Diffraction Theory

Transmitted wave: \[ \Psi_{\text{trans}}(\vec{r}) = \psi_{\text{tr}0} e^{i k_0 \cdot \vec{r}} + \psi_{\text{tr}H} e^{i k_H \cdot \vec{r}} \]

\[
\psi_{\text{tr}0} = \left[ \cos \Phi - \frac{iy}{\sqrt{1 + y^2}} \sin \Phi \right] e^{i(\phi_1 - \phi_0)} A_0
\]

\[
\psi_{\text{tr}H} = \left[ \frac{-iy}{\sqrt{1 + y^2}} \sin \Phi \right] e^{-i(\phi_1 + \phi_0)} A_0
\]

With
\[
\phi_0 = \frac{\nu_0 D}{\cos \theta_B} \quad , \quad \phi_1 = \frac{\nu_H D}{\cos \theta_B}
\]

\[ \Phi = \left( \frac{\nu_H}{\sqrt{1 + y^2}} \right) \frac{D}{\cos \theta_B} \]

Transmitted intensities:
\[
I_0 = \left| \psi_{\text{tr}0} \right|^2 = A_0^2 \left[ \cos^2 \Phi + \frac{y^2}{1 + y^2} \sin^2 \Phi \right]
\]
\[
I_H = \left| \psi_{\text{tr}H} \right|^2 = A_0^2 \left[ \frac{1}{1 + y^2} \sin^2 \Phi \right]
\]
Transmitted Intensities

For the $\langle 111 \rangle$ reflection in Si at $\lambda = 2.70$ Å:

\[
y = 1 \quad \Rightarrow \quad 0.9 \text{ arcsec}
\]
Some Consequences of Dynamical Diffraction

• Pendellösung interference
  \[ \Phi = \left( v_H \frac{1}{\sqrt{1 + y^2}} \right) \frac{D}{\cos \theta_B} \]

• Anomalous transmission

• Angle amplification
Angle Amplification

For small $\delta$ ($\sim 10^{-3}$ arcsec): $\frac{\Omega}{\delta} \approx 10^6$
Practical Neutron Interferometer
4π Rotational Symmetry of Spinors

Rotation operator: \( R_{\hat{n}}(\alpha) = e^{\frac{-i}{\hbar} \hat{n} \cdot \vec{S}} \)

Spin-1/2 particle: \( \vec{S} = \frac{1}{2} \hbar \vec{\sigma} \) so \( R_{\hat{n}}(\alpha) = e^{\frac{-i}{2} \hat{n} \cdot \vec{\sigma}} \)

Rotations about z-axis: \( R_z(\alpha) = \begin{pmatrix} e^{-i\alpha/2} & 0 \\ 0 & e^{i\alpha/2} \end{pmatrix} \)

Symmetry:
\( R_z(2\pi) \chi = -\chi \)
\( R_z(4\pi) \chi = \chi \)
Larmor precession phase:

\[ \Delta \phi = \pm 2\pi \mu_n m_n \lambda B \ell / \hbar^2 \]
4π-Periodicity of the spinor wave function under space rotation

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![Diagram of experimental setup](image)
Quantum Phase Shift Due To Gravity (COW Experiments)

\[ \Delta \phi = \frac{2\pi \lambda gA}{\hbar^2} m_{in} m_{grav} \]

\[ A = H \ell = \text{area of parallelogram} \]

\[ m_{in} = \text{neutron inertial mass} \]

\[ m_{grav} = \text{neutron gravitational mass} \]

test of weak equivalence principle at the quantum limit
$$\Delta \phi_{\text{grav}} = \frac{2\pi \lambda g A_0}{h^2} m_{\text{in}} m_{\text{grav}} \sin \alpha = q \sin \alpha$$

$$A_0 = \text{area of parallelogram at } \alpha = 0$$

measured: $q = 54.3$
theory: $q = 59.6$
Systematic Effects in the COW Experiments

\[ q_{\text{COW}} = \left[ \left( q_{\text{grav}} (1 + \varepsilon) + q_{\text{bend}} \right)^2 + q_{\text{Sagnac}}^2 \right]^{\frac{1}{2}} \]

- Dynamical diffraction correction
- Bending of interferometer
- Earth’s rotation

Sagnac effect: \( \Delta \phi_{\text{Sagnac}} = \frac{2m_{\text{in}}}{\hbar} \vec{\Omega} \cdot \vec{A} \) due to Earth's rotating frame

Bending effect: repeat experiment with x rays, different wavelengths
data from Werner, et al. (1988)

Layer and Greene (1991): x rays do not fill the Borrmann fan as completely as neutrons

Littrell, et al. (1997) results:

<table>
<thead>
<tr>
<th>experiment</th>
<th>q_{COW} theory [rad]</th>
<th>q_{COW} meas. [rad]</th>
<th>discrepancy (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SS, 440</td>
<td>50.97(5)</td>
<td>50.18(5)</td>
<td>-1.6</td>
</tr>
<tr>
<td>SS, 220</td>
<td>100.57(10)</td>
<td>99.02(10)</td>
<td>-1.5</td>
</tr>
<tr>
<td>LLL, 440</td>
<td>113.60(10)</td>
<td>112.62(15)</td>
<td>-0.9</td>
</tr>
<tr>
<td>LLL, 220</td>
<td>223.80(10)</td>
<td>221.85(30)</td>
<td>-0.9</td>
</tr>
</tbody>
</table>

Upcoming new effort (H. Kaiser, S. Werner, FEW, et al.):

Suspend interferometer inside chamber filled with ZnBr$_2$+D$_2$O (floating COW)
Measuring the Neutron's Mean Square Charge Radius Using Neutron Interferometry

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**neutron**: neutral but consists of charged quarks

neutron mean square charge radius:

\[ \langle r_n^2 \rangle = \int \rho(r) r^2 d^3r \]

expected to be negative (positive core, negative skin):

Fermi and Marshall, 1947
Neutron Electric Scattering Form Factor

\[ G_E^n(Q^2) = \text{Fourier transform of neutron charge density (Breit frame)} \]

Expanding in momentum transfer \( Q^2 \):

\[ G_E^n(Q^2) = q_n - \frac{1}{6} \langle r_n^2 \rangle Q^2 + \ldots \]

In the low \( Q^2 \) limit:

\[ \langle r_n^2 \rangle = -6 \left. \frac{dG_E(Q^2)}{dQ^2} \right|_{Q^2 = 0} \]
\( \langle r_n^2 \rangle \) constrains the slope of \( G_E(Q^2) \) in electron scattering experiments and theory (e.g. Bates, Jefferson Lab)

Neutron-Atom Coherent Scattering Length

\[ b_{\text{coh}} = b_N + Z \left[ 1 - f(q) \right] b_{ne} \]

Fourier transform of charge density

\[ f(q) = \frac{1}{\sqrt{2\pi}} \int e^{iq \cdot r} \rho_{\text{atom}}(r) d^3r \]

\[ b_{ne} = \text{neutron-electron scattering length} \]

In 1\textsuperscript{st} Born approximation:

\[ \langle r_n^2 \rangle = 3a_0 \left( \frac{m_e}{m_n} \right) b_{ne} = (86.34 \text{ fm}) b_{ne} \]
Foldy Scattering Length

\[ b_F = -\frac{\gamma e^2}{2m_e c^2} = -1.468 \times 10^{-3} \text{ fm} \]

Incorrect interpretation: \( b_{ne\ (\text{meas.})} = b_{\text{intrinsic}} + b_F \)

Correct interpretation: The experimentally measured value of \( b_{ne} \) is entirely due to the static charge distribution in the neutron.
### Previous Experiments

**TABLE I.** Experimental results of $b_{ne}$ in units of $10^{-3}$ fm.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Target</th>
<th>Result</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angular scattering</td>
<td>Ar</td>
<td>$-0.1 \pm 1.8$</td>
<td>1947 [7] Fermi</td>
</tr>
<tr>
<td>Transmission</td>
<td>Bi</td>
<td>$-1.9 \pm 0.4$</td>
<td>1951 [8] Havens</td>
</tr>
<tr>
<td>Angular scattering</td>
<td>Kr, Xe</td>
<td>$-1.5 \pm 0.4$</td>
<td>1952 [9] Hamermesh</td>
</tr>
<tr>
<td>Mirror reflection</td>
<td>Bi/O</td>
<td>$-1.39 \pm 0.13$</td>
<td>1953 [10] Hughes</td>
</tr>
<tr>
<td>Angular scattering</td>
<td>Kr, Xe</td>
<td>$-1.4 \pm 0.3$</td>
<td>1956 [11] Crouch</td>
</tr>
<tr>
<td>Crystal spectrometer transmission</td>
<td>Bi</td>
<td>$-1.56 \pm 0.05$</td>
<td>1959 [2] Melkonian</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$-1.49 \pm 0.05$</td>
<td>1976 in Ref. [15]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$-1.44 \pm 0.033 \pm 0.06$</td>
<td>1997 this work</td>
</tr>
<tr>
<td>Angular scattering</td>
<td>Ne, Ar, Kr, Xe</td>
<td>$-1.34 \pm 0.03$</td>
<td>1966 [12] Krohn</td>
</tr>
<tr>
<td>Angular scattering</td>
<td>Ne, Ar, Kr, Xe</td>
<td>$-1.30 \pm 0.03$</td>
<td>1973 [13] Krohn</td>
</tr>
<tr>
<td>Single crystal scattering</td>
<td>$^{186}$W</td>
<td>$-1.60 \pm 0.05$</td>
<td>1975 [14] Alexandrov</td>
</tr>
<tr>
<td>Filter-transmission, mirror reflection</td>
<td>Pb</td>
<td>$-1.364 \pm 0.025$</td>
<td>1976 [15] Koester</td>
</tr>
<tr>
<td>Filter-transmission, mirror reflection</td>
<td>Bi</td>
<td>$-1.393 \pm 0.025$</td>
<td>1976 [15] Koester</td>
</tr>
<tr>
<td>$n$-TOF transmission, mirror reflection</td>
<td>Bi</td>
<td>$-1.55 \pm 0.11$</td>
<td>1986 [16] Alexandrov</td>
</tr>
<tr>
<td>Filter-transmission, mirror reflection</td>
<td>Pb, Bi</td>
<td>$-1.32 \pm 0.04$</td>
<td>1986 [17] Koester</td>
</tr>
<tr>
<td>$n$-TOF transmission</td>
<td>thorogenic $^{208}$Pb</td>
<td>$-1.31 \pm 0.03 \pm 0.04$</td>
<td>1995 [1] Kopecky</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$-1.33 \pm 0.027 \pm 0.03$</td>
<td>1997 this work</td>
</tr>
<tr>
<td>Filter-transmission, mirror reflection</td>
<td>Pb-isotopes, Bi</td>
<td>$-1.32 \pm 0.03$</td>
<td>1995 [5] Koester</td>
</tr>
<tr>
<td>Garching-Argonne compilation</td>
<td>[12,13,15,17]</td>
<td>$-1.31 \pm 0.03$</td>
<td>1986 [3] Sears</td>
</tr>
<tr>
<td>Dubna compilation</td>
<td>[14,16]</td>
<td>$-1.59 \pm 0.04$</td>
<td>1989 [19] Alexandrov</td>
</tr>
<tr>
<td>Foldy approximation, $b_F$</td>
<td></td>
<td>$-1.468$</td>
<td>1952 [18] Foldy</td>
</tr>
</tbody>
</table>
Neutron Interferometer Experiment

off Bragg: \[ b_{\text{coh}} = b_N + Z[1 - f(0)]b_{\text{ne}} = b_N \]

near Bragg: \[ b_{\text{coh}} = b_N + Z[1 - f(\bar{H}_{111})]b_{\text{ne}} \]
Dynamical Phase Shift Through Bragg

\[ \Delta \Phi_{\text{dyn}} = \frac{\nu_H}{\cos \theta_B} \left( y \pm \sqrt{1 + y^2} \right) D \]

\[ D = \text{crystal thickness} \]

scaled misset angle \[ y = \frac{k \sin 2 \theta_B}{2 \nu_H} \]

\[ \nu_H = \frac{F_{111} \lambda}{V_{\text{cell}}} = \frac{\sqrt{32} \lambda}{V_{\text{cell}}} b_{\text{coh}} \]

near Bragg: \[ b_{\text{coh}} = b_N + Z[1 - f(H_{111})] b_{\text{ne}} \]
What we must measure:

1. Net dynamical phase shift through Bragg $\nu_H \rightarrow b_N + Z[1 - f(\vec{H}_{111})]b_{ne}$ to $\sim 10^{-5}$

   The maximum slope is $\sim 88\pi$/arcsec so we need 0.01 arcsec angular precision to detect every $2\pi$ of phase shift

2. Forward phase shift off Bragg $\rightarrow b_N$ to $\sim 10^{-5}$ and subtract

3. Neutron wavelength to $\sim 10^{-3}$

4. Calculate $f(\vec{H}_{111})$ to $\sim 10^{-3}$

This will give $b_{ne}$, and hence $\langle r_n^2 \rangle$, to $< 1\%$
Tulane-NIST neutron charge radius experiment

10 cm lever with nylon flexure bearing

Physik Instrumente P-753 PZT nanopositioner
25 µm range
1.0 nm precision (.002 arcsec)

Four Micro-E mercury rotation encoders
.010 arcsec precision
Preliminary Data:

These data were taken at NIST in September 2005.
The Neutron
by Gina Berkeley

When a pion an innocent proton seduces
With neither excuses
Abuses
Nor scorn
For its shameful condition
Without intermission
The proton produces: a neutron is born.

What love have you known
O neutron full grown
As you bombinate into the vacuum alone?

Its spin is 1/2, and its mass is quite large
-about 1 AMU
but it hasn’t a charge;
Though it finds satisfaction in strong interaction
It doesn’t experience Coulombic attraction

But what can you borrow
Of love, joy, or sorrow
O neutron, when life has so short a tomorrow?

Within its
Twelve minutes
Comes disintegration
Which leaves an electron in mute desolation
And also another ingenuous proton
For other unscrupulous pions to dote on.
At last, a neutrino;
Alas, one can see no
Fulfilment for such a leptonic bambino.
No loving, no sinning
Just spinning and spining
Eight times through the globe without ever beginning...
A cycle mechanic
No anguish or panic
For such is the pattern of life inorganic.

O better
The fret a
Poor human endures
Than the neutron’s dichotic
Robotic
Amours.