Systematics in neutron lifetime experiments
(Intricacies of Monte Carlo simulations for non-magnetic UCN storage)

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About simulations, especially in connection with the Gatchina lifetime experiment (Serebrov et al., Phys. Lett. B 605 (2005) 72)
Gravitational UCN storage system from Gatchina (here used with LTF)

- 1 – neutron guide from UCN turbine;
- 8 – UCN storage trap;
- 12 – UCN detector;
- 14 – evaporator.
- 1-8-12 – entrance/exit channel
The good news first

• The oil at -160 C behaved very well → low loss and extremely large storage lifetime

\[ \tau_{st} \equiv 870 \text{ s} \]

(only 2% away from data group lifetime value)
Typical measuring cycle

1. filling 160 s (time of trap rotation (35 s) to monitoring position is included);
2. monitoring 300 s;
3. holding 300 s or 2000 s (time of trap rotation (7 s) to holding position is included);
4. emptying has 5 periods 150 s, 100 s, 100 s, 100 s, 150 s (time of trap rotation (2.3 s, 2.3 s, 2.3 s, 3.5 s, 24.5 s) to each position is included);
5. measurement of background 100 s.

\[
N(t_2) = N(t_1) \cdot \exp \left( -\frac{t-t_1}{\tau_{st}} \right)
\]

\[
\tau_{st} = \frac{t_2-t_1}{\ln(\frac{N(t_1)}{N(t_2)})}
\]
Uncertainties in MC simulations of non-magnetic UCN storage

- Initial spectrum
- How can we simplify complex geometries?
- Surface conditions – impurities, roughness
- ...
Cold neutron and UCN reflection from rough surfaces (Steyerl 1972, Ignatovich 1973, Sinha et al., 1988 etc.)

- Angular scattering distribution depends on roughness parameters (mean amplitude, mean surface slope …). Usually far from “totally diffuse”, especially for smooth oil.
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- (forgotten): Effect of roughness on loss coefficient $\mu$ (Ignatovich (1973) and his book). Roughness may reduce $\mu$!
Rel. change of loss coefficient $\mu$ for micro-roughness with $\alpha = 1 \text{ mrad}, w = 2\mu\text{m}, k_{c_0} = 0.0802 \text{ nm}^{-1}$; exact vs. approx. (25)
Cold neutron and UCN reflection from rough surfaces (Steyerl 1972, Ignatovich 1973, Sinha et al., 1988 etc.)

- (forgotten): Effect of roughness on loss coefficient $\mu$ (Ignatovich (1973) and his book)
- Integral probability of diffuse scattering: dependence on UCN energy, angle of incidence, roughness parameters. Never a constant!
Scattered intensity for micro-roughness with $\alpha = 1$ mrad, $w = 2\mu$m, $k_{c0} = 0.0802$ nm$^{-1}$; exact vs. approximation (24)

For $k/k_{c0} =$
- 0.2: exact
- 0.6: exact
- $1 - 10^{-4}$: exact

$\rho_{D0}$ vs. $\theta_{i}[^\circ]$ (measured from normal)
Cold neutron and UCN reflection from rough surfaces (Steyerl 1972, Ignatovich 1973, Sinha et al., 1988 etc.)

- Integral probability of diffuse scattering: dependence on UCN energy, angle of incidence, roughness parameters. Never a constant!

- Perturbation must be taken to **second order** to get a full picture (specular reflection; effects of loss coefficient $\eta$).
Angular distribution of roughness scattering
(Ignatovich’s book)

• Must satisfy detailed balance requirement:

\[ I_{sc}(\Omega_i \rightarrow \Omega) = S(\Omega_i, \Omega) \cos \theta \]

\[ (\cos \theta = \text{Lambert factor}) \]

where \( S(\Omega_i, \Omega) \) is symmetric in \((\theta, \theta_i)\) and in \((\varphi, \varphi_i)\). For instance

\[ I_{sc}(\Omega_i \rightarrow \Omega) = (\cos \theta_i \cos \theta) \cos \theta \quad \checkmark \]

However: \( I_{sc}(\Omega_i \rightarrow \Omega) = \text{const.} \quad \times \)

(violates micro-reversibility, time reversal invariance, cannot establish or maintain an isotropic equilibrium distribution)

\( \cos \theta \) is needed also for generation of initial isotropic distribution
Scope and techniques of my MC simulation

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• Would not know how to approximate the complex entrance/exit channel by a simple geometry. (Can the same roughness model be used for trap and channel?)
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• For maximum consistency I used the same initial spectrum [with a heuristic factor \( \exp(-h_0/h_{sp}) \)], \( \tau_n = 878.5 \text{s} \) and \( \eta = 2 \times 10^{-6} \).
Scope and techniques of my MC simulation

• For maximum consistency I used same initial spectrum and $\tau_n = 878.5$ s, $\eta = 2 \times 10^{-6}$.

• Implementation of realistic roughness scattering distribution $I_{sc}(\Omega_i \rightarrow \Omega)$ in MC simulations needs tabulation, and is slow and not trivial:

  (a) mapping; (b) “trial and error”
Scope and techniques of my MC simulations

- Implementation of realistic roughness scattering distribution $I_{sc}(\Omega_i \rightarrow \Omega)$ in MC simulations needs tabulation, and is slow and not trivial:
  - (a) mapping; (b) “trial and error”
- Loss coefficient correction also requires tabulation.
Simulation for a narrow cylindrical trap; no coupling; w/ decay due to $\tau_n$ and theor. wall loss for $\eta = 2 \times 10^{-6}$.

- Short storage time (300 s)
- Long storage time (2000 s)
Published measurement of Gatchina count-rates and simulation for short and long storage

My simulation of the same, but without entrance/exit channel
Effect of “exit channel” on counting rates:

- Publ. data and sim’s (including “exit channel”)
- My sim. (trap only) for 0% diffuse
- My sim. (trap only) for ~10% diffuse (on average)
“Soft roughness”: slow relaxation toward isotropic UCN distribution

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• “Soft roughness”: Small mean-square surface slope ($\alpha \approx 10^{-3}$);

• Models used:
  
  (a) Gaussian correlation function:
  \[
  \langle \xi(\mathbf{r})\xi(\mathbf{r}+\mathbf{\delta}) \rangle_A = b^2 \exp[-\frac{\delta^2}{2w^2}]
  \]

  (b) “$K_0$-model” for liquids or glasses:
  \[
  \langle \xi(\mathbf{r})\xi(\mathbf{r}+\mathbf{\delta}) \rangle_A \sim K_0\left\{\frac{((\delta^2+\delta_0)^2/(2w^2))^{1/2}}{2}\right\}
  \]
“Soft roughness”: slow relaxation toward isotropic UCN distribution

- Models used:
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  \langle \xi(\rho)\xi(\rho+\delta) \rangle_{\text{Area}} = b^2 \exp[-\delta^2/(2w^2)]
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  (b) “\(K_0\)-model” for liquids or glasses:
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  \langle \xi(\rho)\xi(\rho+\delta) \rangle_A \sim K_0\left\{ [(\delta^2+\delta_0)^2/(2w^2)]^{1/2} \right\}
  \]

- For mean square slope \(\alpha^2 = 2b^2/w^2 \ll 1\) and high symmetry, there are quasi-stationary orbits w/ motion mainly along cylinder axis.
The case of purely specular reflection (SR)

- SR leads to highly stable orbits in cylindrical trap.

UCN “sliding” along the cylindrical rim and being reflected from vertical side walls
Simulation for a narrow cylindrical trap; no coupling; w/ decay due to $\tau_n$ and wall loss for $\eta=2\times10^{-6}$

- Gauss; $b = 1.0$ nm, $w = 2$ $\mu$m; 300 s
- same for 2000 s; $\Delta \lambda/\lambda = -0.010$
- liquid ($K_o$); $b = 0.5$ nm; $w = 3$ $\mu$m; 300 s
- same for 2000 s; $\Delta \lambda/\lambda = -0.0036$
“Soft roughness”: slow relaxation toward isotropic UCN distribution

- For $\alpha^2 = 2 \frac{b^2}{w^2} << 1$ and a high vessel symmetry, there are quasi-stationary orbits w/ motion mainly along cylinder axis.
- $\Rightarrow$ Slow random walk away from the quasi-stationary orbits toward isotropy
  $\rightarrow$ slow spill-over during long storage.
“Soft roughness”: slow relaxation toward isotropic UCN distribution

- \( \Rightarrow \) Slow random walk away from the quasi-stationary orbits toward isotropy
  \( \rightarrow \) slow “spill-over” during long storage.

- If “spill-over” is within fluctuations of real detector background, it may go unnoticed.
“Soft roughness”: slow relaxation toward isotropic UCN distribution

• If “spill-over” is within fluctuations of real detector background, it may go unnoticed.
• These UCN are missing from UCN counts for the long storage time
  ⇒ which requires a significant correction to calculated storage lifetimes for spectral interval #1 (and other intervals, too?).
Simulation for a narrow cylindrical trap; no coupling; w/ decay due to \( \tau_n \) and wall loss for \( \eta = 2 \times 10^{-6} \)

- **Gauss;** \( b = 1.0 \text{ nm}, w = 2 \mu \text{m}; 300 \text{ s} \)
- **same for** 2000 s; \( \Delta \lambda / \lambda = -0.010 \)
- **liquid (K)\()**; \( b = 0.5 \text{ nm}, w = 3 \mu \text{m}; 300 \text{ s} \)
- **same for** 2000 s; \( \Delta \lambda / \lambda = -0.0036 \)

**Graph:**
- **Y-axis:** Simulated counts (+1) per 20s and \( 10^6 \) runs
- **X-axis:** Time measured from start of monitoring [s]
“Soft roughness”: slow relaxation toward isotropic UCN distribution

- These UCN are missing from UCN counts for the long storage time
  ⇒ which requires a significant correction to calculated storage lifetimes for spectral interval #1 (and other intervals, too?).

- Relaxation process is fairly slow even for “steep roughness” ($\alpha \approx 1$).
  ⇒ mixing of spectral intervals
Simulated counts (+1) per 20s and $10^6$ runs

- Red, dashed line: Gauss; $b = 1.0$ nm, $w = 2 \, \mu$m; 300 s
- Black line: same for 2000 s; $\Delta \lambda/\lambda = -0.010$
- Blue, dashed line: liquid ($K_0$); $b = 0.5$ nm; $w = 3 \, \mu$m; 300 s
- Green line: same for 2000 s; $\Delta \lambda/\lambda = -0.0036$
- Magenta, dashed line: liquid ($K_0$); $b = 2$ nm; $w = 4$ nm; 300 s
- Cyan line: same for 2000 s; $\Delta \lambda/\lambda = -0.026$

Time measured from start of monitoring [s]
“Soft roughness”: slow relaxation toward isotropic UCN distribution

• Relaxation process is fairly slow even for “steep roughness” \((\alpha \approx 1)\).
  \[ \Rightarrow \text{mixing of spectral intervals} \]

• \[ \Rightarrow \] The five spectral intervals are not well defined.
  \[ \Rightarrow \] The five \(\langle \gamma \rangle\)-values show an “erratic” behavior and up to 10% difference between short and long storage; … up to 20% between \(\alpha \approx 1\) and \(\alpha \ll 1\).
\langle \gamma \rangle \text{ from simulations for a narrow trap}

Liquid $K_0$-model for $\eta = 2 \times 10^{-6}$
smooth roughness ($b = 0.5 \text{ nm}, w = 3 \mu\text{m}$)

- ■ short storage time (300s)
- ● long storage time (2000s)

\langle \gamma \rangle \text{ [1/s] for jagged roughness ($b = 2\text{nm}, w = 4\text{nm}$)}
“Soft roughness”: slow relaxation toward isotropic UCN distribution

- The five spectral intervals are not well defined.
- The five $\langle \gamma \rangle$-values show an “erratic” behavior and up to 10% difference between short and long storage; … up to 20% between $\alpha \approx 1$ and $\alpha << 1$.
- For purely specular reflection, $\langle \gamma \rangle_1$ to $\langle \gamma \rangle_5$ do not even show the “correct” sequence.
Attempt of extrapolation to $1/\tau_n$ for $\gamma \to 0$
for cylindrical trap and specular reflection

$\tau_n = 878.5 \text{ s} \rightarrow \text{Gatchina}$

$\tau_n = 885.7 \text{ s} \rightarrow \text{Data group}$
“Soft roughness”: slow relaxation toward isotropic UCN distribution

• For purely specular reflection, $\langle \gamma \rangle_1$ to $\langle \gamma \rangle_5$ do not even show the “correct” sequence.

• Spectral mixing is also evident from the mean spectral energy change, $\Delta h$, between short and long storage.
Spectral changes for the five counting intervals

Liquid $K_0$-model

Simulation for $\eta = 2 \times 10^{-6}$, $b = 0.5$ nm, $w = 3$ $\mu$m
for full spectrum: $(\Delta \langle h \rangle)/\langle h \rangle = 0.6 \pm 0.2$ %
“Soft roughness”: slow relaxation toward isotropic UCN distribution

• Spectral mixing is also evident from the mean spectral energy change, \( \Delta h \), between short and long storage.

• Spectral changes between short and long storage imply a change of UCN detection efficiency of order 1%. 
Conclusions

• Specifically for the Gatchina experiment: In their extrapolation to obtain $\tau_n$ both the $x$-values ($\gamma$) and the $y$-values ($1/\tau_{st}$) are uncertain.
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• Specifically for the Gatchina experiment: In their extrapolation to obtain $\tau_n$, both the $x$-values ($\gamma$) and the $y$-values ($1/\tau_{st}$) are uncertain.

• More generally on MC simulations as a basis of data interpretation: Far from being a panacea, MC simulations tend to be riddled with uncertainties regarding models, input parameters, “loss factors”, numerical instabilities …
“Lifetime Conclusion” of Boris Yerozolimsky:

“If you try to improve the $\tau_n$-value to the level $\sim 10^{-3}$ or better you will run against a brick wall of exponentially growing problems.”
Thank you
Transparency of LTF film for isotropic incidence

\[ 0.1 \times \frac{\Delta \langle \varepsilon_0 \rangle}{\langle \varepsilon_0 \rangle} / \frac{\Delta h}{h_0} \]

use \( h_0 = 1.04 \, \text{m} \) for LTF at room temperature
Transparency of Al window for isotropic incidence

0.1 times \( \frac{\Delta \langle \varepsilon_D \rangle}{\langle \varepsilon_D \rangle} \left[ \frac{\Delta h}{h_0} \right] \)

Use \( h_c = 0.52 \text{ m} \) for Al
An ‘accordion-like’ UCN storage system

**Features:** (a) Use a bellows with horizontal axis.
An ‘accordion-like’ UCN storage system

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(b) **Make volume** $V$ **variable within a large factor of** $\sim 5-10$. 
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(c) Surface area and its distribution over height are constant.
**An ‘accordion-like’ UCN storage system**

**Features:**
(a) Use a bellows with horizontal axis.
(b) Volume $V$ variable within a large factor of $\sim 10$.
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(d) Times for loading, holding, emptying are scaled $\sim V$.

$\Rightarrow$ Spectral development is the same for different $V$. 

![Diagram of an 'accordion-like' UCN storage system with specifications and features.](image-url)
An ‘accordion-like’ UCN storage system

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(a) Use a bellows with horizontal axis.  
(b) Volume $V$ variable within a large factor of $\sim 10$.  
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(d) Times for loading, holding, emptying are scaled $\sim V$  
   $\Rightarrow$ Spectral development is the same for different $V$  
(e) Provide up-down symmetry and ensure that all UCN have enough energy to reach roof  $\Rightarrow$ cut spectrum from below
An ‘accordion-like’ UCN storage system

**Calculated characteristics:**

(a) Dependence of $\tau_{st}^{-1}$ on wall collision frequency is highly linear.

*Fitting function: $y = \alpha x + 1000/\tau$*

Points 1-4: $\tau = 885.75(2e^{-4})$, $\alpha = 9.19(4e^{-6})$

Points 1-10: $\tau = 885.82(3e^{-2})$, $\alpha = 9.19(7e^{-5})$

Points 7-10: $\tau = 886.01(5e^{-2})$, $\alpha = 9.19(7e^{-5})$

Efflux constant $\gamma_e [1/s]$, calculated from count-rates
An ‘accordion-like’ UCN storage system

*Calculated characteristics (continued):*

(b) The linearity holds for any UCN spectrum.
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*Calculated characteristics (continued):*

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(e) The “loading/emptying” correction of MAMBO is also reduced.
An ‘accordion-like’ UCN storage system

Calculated characteristics (continued):

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(c) The linearity holds for any energy dependence of UCN reflectivity.
(d) No “gravity correction” is required.
(e) The “loading/emptying” correction of MAMBO is strongly reduced.
(f) Extrapolation to the neutron lifetime is strictly linear and reliable.
An ‘accordion-like’ UCN storage system

*Calculated characteristics (continued)*:

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(c) The linearity holds for any energy dependence of UCN reflectivity.
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(e) The “loading/emptying” correction of MAMBO is strongly reduced.
(f) Extrapolation to the neutron lifetime is strictly linear and reliable.

(g) Last but not least: complex geometry ensures isotropic UCN distribution even for purely specular reflections.
Present status:

A better bellows has been made in Dubna. Mechanical systems, cryogenics and vacuum have been upgraded. When ready?
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Hopefully, soon.