Neutron Lifetime Measurements
Some problems with gravo-magneto traps

David Bowman
ORNL
N lifetime data from PDG

\section*{\textit{n} MEAN LIFE}

We now compile only direct measurements of the lifetime, not those inferred from decay correlation measurements. For the average, we only use measurements with an error less than 10 s.

The most recent result, that of SEREBROV 05 (for a more detailed account, see SEREBROV 08A), is so far from other results that it makes no sense to include it in the average. It is up to workers in this field to resolve this issue. Until this major disagreement is understood our present average of 885.7 ± 0.8 s must be suspect.

For recent reviews of neutron physics, see NICO 05A and SEVERIJNS 06.

Limits on lifetimes for \textit{bound} neutrons are given in the section "\textit{p} PARTIAL MEAN LIVES."

<table>
<thead>
<tr>
<th>VALUE (s)</th>
<th>DOCUMENT ID</th>
<th>TECN</th>
<th>COMMENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>885.7 ± 0.8</td>
<td>OUR AVERAGE</td>
<td></td>
<td></td>
</tr>
<tr>
<td>886.3 ± 1.2 ± 3.2</td>
<td>NICO 05</td>
<td>CNTR</td>
<td>In-beam \textit{n}, trapped \textit{p}</td>
</tr>
<tr>
<td>885.4 ± 0.9 ± 0.4</td>
<td>ARZUMANOV 00</td>
<td>CNTR</td>
<td>UCN double bottle</td>
</tr>
<tr>
<td>889.2 ± 3.0 ± 3.8</td>
<td>BYRNE 96</td>
<td>CNTR</td>
<td>Penning trap</td>
</tr>
<tr>
<td>882.6 ± 2.7</td>
<td>MAMPE 93</td>
<td>CNTR</td>
<td>Gravitational trap</td>
</tr>
<tr>
<td>888.4 ± 3.1 ± 1.1</td>
<td>NESVIZHEV... 92</td>
<td>CNTR</td>
<td>Gravitational trap</td>
</tr>
<tr>
<td>887.6 ± 3.0</td>
<td>MAMPE 89</td>
<td>CNTR</td>
<td>Gravitational trap</td>
</tr>
<tr>
<td>891 ± 9</td>
<td>SPIVAK 88</td>
<td>CNTR</td>
<td>Beam</td>
</tr>
</tbody>
</table>
New approach: gravo-magneto traps

• Gravo-magneto traps don’t have material walls.
• G-M traps can be fully understood and modeled. We are dealing with magnetic fields, magnetic moments, gravity, and F=Ma, not surface physics.
• I will discuss some problems.
Problems for G-M traps

• Spin-flip losses
• Serebrov measurement and chaos.
• Cleaning the trap – elimination of quasi-bound orbits
• Effect of mixed (chaotic + regular) orbits
  • Difficulties
  • Possible solutions
Majorana 1932

- Polarization flips if a neutron passes through a field zero
- “Reference” calculation
- Calculated the spin-flip probability of a polarized atom passing near a minimum of $B$.
  - Assumed constant velocity along a classical trajectory and treated the spin quantum mechanically.

\[
P_{\text{flip}} \sim \exp \left[ -\frac{\pi}{2} \eta \right]
\]

\[
\eta = \frac{\mu B}{\hbar} / \frac{d \mathcal{I}}{dt}
\]
Majorana’s expression can’t be directly applied to magnetic traps

- High-order multipole field traps don’t have minima.
- Majorana assumed regions of constant field to prepare and analyze the spin.
- In a trap the velocity is not constant. The spin doesn’t commute with the Hamiltonian.
- Treat both spin and position quantum mechanically.
Current is in the y direction in the x-y plane. Neutron moves along moves along z.
Schrodinger’s eq. in system with $z$ along B and dimensionless coordinates

trapped spin

$$- \left( \frac{d^2 \psi}{dx^2} + \frac{d \chi}{dx} \frac{1}{\sqrt{1 + \text{Exp}[-2x] \sqrt{1 + \text{Exp}[2x]}}} \right) - 2\xi \left( \delta - \sqrt{1 + \text{Exp}[-2x]} \right) \psi = 0$$

free spin

$$- \left( \frac{d^2 \psi}{dx^2} - \frac{d \chi}{dx} \frac{1}{\sqrt{1 + \text{Exp}[-2x] \sqrt{1 + \text{Exp}[2x]}}} \right) - 2\xi \left( \delta + \sqrt{1 + \text{Exp}[-2x]} \right) \chi = 0$$

$$\xi = \frac{m \lambda^2 \mu B_y}{\hbar^2}$$

$$\delta = \frac{E_n}{\mu B_y}$$
Potentials for $\sigma \cdot B = \pm$

En, bound and unbound potentials
Use WKB approximation

- Solve Schrödinger’s eq. for trapped spin, $\psi$, neglecting coupling.
- Insert $\psi$ into unbound eq. and calculate $\chi$.
- $P_{\text{flip}} \sim \exp\left[-\frac{\pi}{2} \eta\right]$

$$\eta = \frac{\mu B_y}{v/\lambda}$$

$\lambda = \text{Halbach period}/2\pi$

$v = \text{Velocity at the bottom of the trap potential}$

for Los Alamos trap $\eta = 2.5 \times 10^5$
Serebrov et al.
• The surfaces of the traps are coated with oil – thickness 15 μm. The surface figure and roughness are not discussed. (Turning produces a roughness from 25 to .025 μm.)

• The assumption used in simulation is that scattering is random (ε) or mirror like (1-ε). No data are presented to support the form of the scattering law. .1<ε<1

• The quasi-spherical trap is a figure of rotation. Angular momentum is conserved. Even when the traps are tilted to allow the neutrons to escape, the potential retains some symmetry. In the absence of randomizing collisions, there are quasi trapped orbits. It is possible that the yield curves depend on quasi-trapped, regular, and cahotic orbits.
Chaos

• Books:
  – “Chaos in Dynamical Systems”, Ott
  – “Regular and Chaotic Dynamics”, Lichtenberg and Lieberman

• Typical behavior of dynamical systems depends on a parameter, in our case En. For small En the motion is regular. The potential is quadratic near the minimum of the potential and the neutron has simple harmonic motion in three dimensions. As the energy is increased chaotic motion may occur.
A test for chaotic motion is provided by “Lyapunov exponents”. $X[t|x_0]$ is a test trajectory. Change the initial conditions, $x_0$, by a small amount and calculate $X_1[t|x_0+dx]$. If $X_1$ and $X$ differ exponentially, then the trajectory is chaotic. Periodic trajectories are useful because one does not have to integrate the trajectory for long times to calculate the Lyapunov exponents. The set of periodic trajectories is dense in phase space.

There are many 2 and 3 dimensional billiards (hard reflecting surfaces) that are chaotic but, there is no Hamiltonian system described by a continuous potential which is known to have only chaotic trajectories.
• Walstrom et al. Simulated the Los Alamos trap. The goal was to determine if quasi-bound orbits could be eliminated by placing an absorber at some height. The trap was filled with mono energetic neutrons. They found an exponential distribution of escape times, but regular orbits remained. These orbits could be eliminated by lowering the absorber below $h=En/g$.

• I interpret this to indicate that the trap had a mixture of chaotic and regular orbits.
A problem

• Experiments and simulations show that quasi-bound trajectories escape with times that are comparable to the neutron lifetime.
• Imagine cleaning the neutron distribution in a trap. Neutrons will remain that have both regular orbits and chaotic.
• In any event, the phase space occupied by neutrons of each energy is not uniform. This phase space does not evolve into itself
• In general, the detection efficiency for decaying neutrons is not uniform over phase space. (For example, the NIST trap.) As the non-uniform distribution evolves with time, the decay rate may be modulated.

• Some possible solutions:
  – Insure that the phase space distribution is uniform for each neutron energy. (One idea would be to temporarily insert a reflecting material blade that makes the trap into a chaotic billiard.)
  – Make the detection efficiency uniform over phase space.
  – Demonstrate using simulation techniques, that the fraction of orbits that are regular is negligibly small.
Difficulties

• It is difficult to simulate chaotic trajectories, because they depend exponentially on initial conditions and round-off errors. (See http://arxiv.org/abs/0706.1494 for a novel approach)

• A standard analog simulation appears difficult because $10^9$ trajectories of 1000 sec each would be required for $dT/T \sim 10^{-4}$. 
Conclusions

• Present situation for the lifetime needs resolution. Lifetime measurements using G-M traps are an opportunity

• Spin-flip losses in magnetic traps are very small

• The behavior of neutron orbits in traps may introduce systematic uncertainties that are difficult to calculate and/or measure.