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FROM ULTRACOLD ANTINEUTRONS TO NEUTRON SPIN ECHO – THE BOOTSTRAP OFFERS SIGNIFICANT GAINS IN SENSITIVITY

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We describe a "bootstrap" method for amplifying the results of certain experiments and discuss its possible application to the search for n–n̄ oscillations using ultracold neutrons (UCN) and to a high resolution neutron resonance spin echo spectrometer. In both these applications the particle "spin" is used to measure an angle defined by external conditions and in these circumstances the "bootstrap" can amplify the resulting effect.

1. Introduction

In this paper we describe what we have called a "bootstrap" technique, which can be used to increase the sensitivity of a class of experiments where the quantity to be measured is represented by the direction of the effective field.

The bootstrap consists of successive rotations about two different centers. Referring to fig. 1, a spin initially pointing along the z axis (point 0 in fig. 1) will, by means of successive 180° precessions alternately around D and E, continually increase its angle with the z axis, so that if θ_0 is the angle between D and the z axis the spin will successively reach angles of $2\theta_0$, $-4\theta_0$, $6\theta_0$, $-8\theta_0$, etc. This idea, first introduced by Yoshiki [1] was developed by Yoshiki and Golub [2] as a means of increasing the sensitivity of the search for neutron–antineutron oscillations using ultracold neutrons (UCN) [3]. Re-invented independently by Gähler, the idea has been applied to a proposed high-resolution neutron spin echo spectrometer [4], which is now under construction.

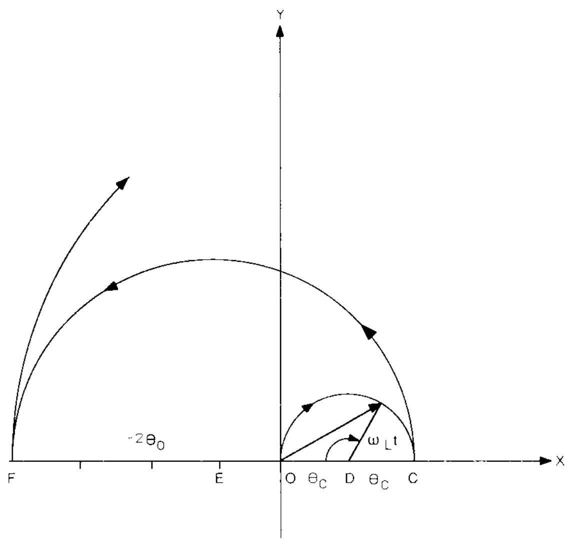

Fig. 1. Bootstrap motion. Consecutive rotations by π rad around alternating centers D and E lead to an ever increasing distance from the origin.

In this paper we would like to call attention to the fact that the bootstrap idea can be useful in two such widely different fields of physics and perhaps stimulate the reader to suggest further applications. As will be seen, the bootstrap is useful in increasing the sensitivity of experiments where the measured quantity can be represented by the direction of the effective field. In the contrary case, where one desires to measure a small precession of the neutron spin as in the search for a neutron electric dipole moment [5], or parity-violating spin precessions [6], the bootstrap is not useful. We will begin with a discussion of the spin echo, as in this case the effective spin is the real spin, so the physical system is likely to be more familiar to most readers.

2. Neutron spin echo

Neutron spin echo (NSE), first proposed by Mezei [7], is based on the idea that the total precession angle of the neutron spin in a known magnetic field is a measure of the time spent by a neutron in that field – and that, by passing through two regions with equal, but antiparallel fields, the precession angle can be set to zero for all neutrons whose velocity does not change in between the two regions. Hence any nonzero precession angle will be a direct measure of the velocity change between the two field regions.

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3. Neutron resonance spin echo

As described above in NSE, the flight time of the neutrons is determined by their angle of precession in a magnetic field. In NRSE [8] the neutron spins do not precess as their trajectories are located in regions of zero, or very small, magnetic field. The flight time is measured by comparing the direction of the neutron spin with a rotating magnetic field whose phase serves as a clock. The comparison is carried out by means of magnetic resonance π coils. In these coils there is a dc magnetic field B_0, along the z axis, and a field $B_r(t)$, rotating in the x–y plane with frequency ω.

On resonance, $\omega = \omega_0 = \gamma B_0$ and the field B_r rotates at exactly the same rate as the Larmor precession of the spin.

As seen from fig. 2 the neutron spin leaves the π coil with its spin at an angle

$$\phi(t_0 + t_\pi) = 2\phi_f(t_0) + \omega_0 t_\pi - \phi(t_0) \tag{1}$$

(with $\phi_f(t_0) = \omega_0 t_0$), with respect to the laboratory x axis. We see from eq. (1) that the spin direction on leaving the coil contains information as to the time of arrival of the neutron at that coil. We now consider an identical pair of such coils separated by a time of flight, T_{AB} (fig. 3).

Then, assuming the neutrons enter A polarized along the x axis ($\phi(t_A) = 0$ for all t_A), application of eq. (1) to coils A and then B yields

$$\phi(t_B + t_\pi) = 2\omega_0(t_A + T_{AB}) + \omega_0 t_\pi - 2\omega_0 t_A - \omega_0 t_\pi = 2\omega_0 T_{AB}. \tag{2}$$

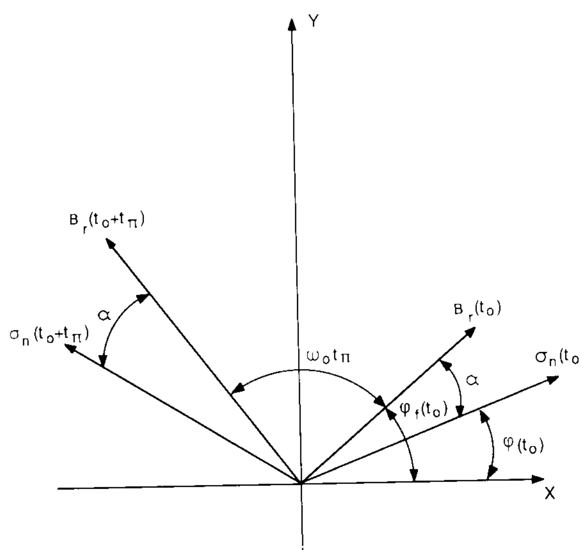

Fig. 2. Action of a π coil. The neutron spin σ_n precesses by π rad around the rotating field B_r. The static field B_0, directed in the z direction, keeps σ_n rotating at the same speed as B_r. The time spent in the coil, t_π, satisfies $\gamma B_r t_\pi = \pi$.

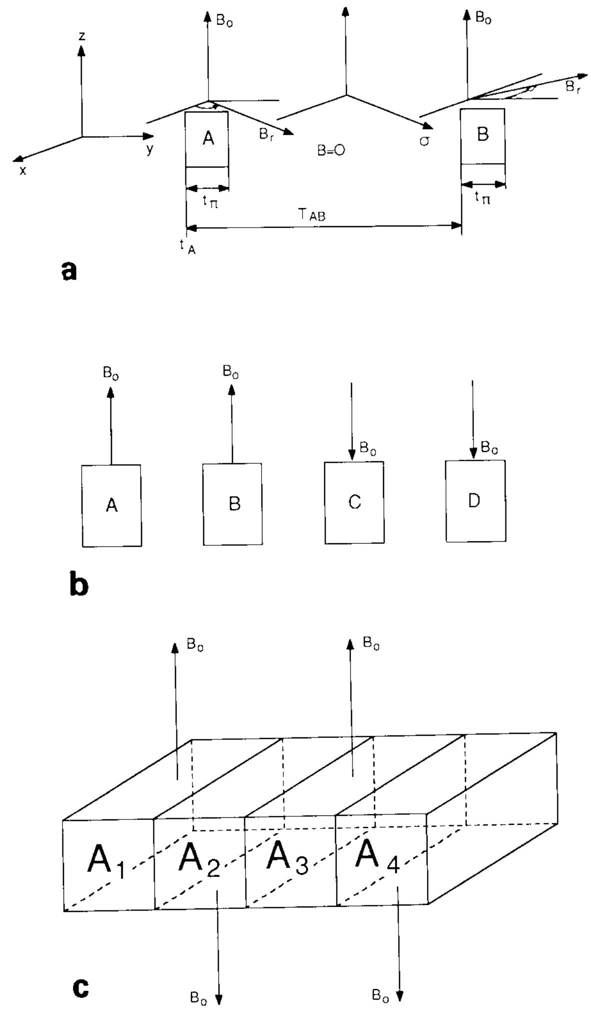

Fig. 3. Outline of an NRSE system. (a) Distribution of magnetic fields in the apparatus. In the intermediate regions $B = 0$ and the neutron spins remain fixed. (b) Direction of dc fields in the π coils. (c) A bootstrap coil with alternating directions of B_0.

If the pair of coils A, B is followed by a second pair C, D with equal but oppositely directed B_0 (and opposite direction of rotation of B_r), we see that on leaving coil D the neutron spin will be pointing in a direction given by

$$\psi(t_D + t_\pi) = 2\omega_0(T_{AB} - T_{CD}), \tag{3}$$

which is directly comparable to the usual spin echo result except for the factor of 2.

4. Bootstrap in NRSE [4]

In the discussion above we have seen how the direction of the neutron spin can be made to carry informa-

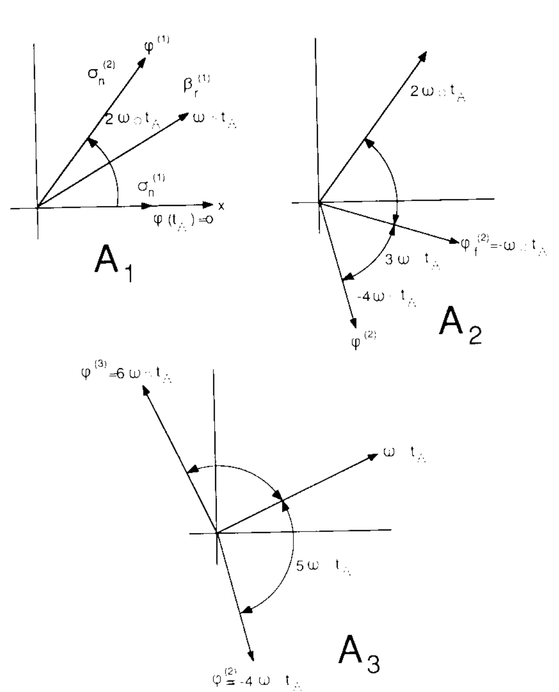

Fig. 4. Operation of the NRSE bootstrap neglecting time spent in the coils. In A_1 the neutrons acquire a phase angle $2\omega_0 t_A$, which is increased in A_2 and A_3 to $-4\omega_0 t_A$ and $6\omega_0 t_A$ respectively.

tion concerning the time of arrival of a given neutron at a coil, as well as the flight time between two coils, encoded in the value of its precession angle, ϕ. We should note that the resolution of an NSE spectrometer depends on the size of the proportionality constant between angle and time in eq. (3). The bootstrap offers the possibility of increasing this proportionality constant and hence the resolution. To simplify the discussion we will neglect the rotation of the system: neutron spin-field B_r, during the time interval t_π that the neutron spends in the π coils. We now consider that each coil in the NRSE system A, B, C, D (fig. 3b) is replaced by a series of coils $A_1, A_2, \ldots, A_N$, etc. with alternating directions of dc field and rotation of B_r (fig. 3c), so that there is always a resonance condition in each field. Now (see fig. 4) a neutron spin entering coil A with $\phi(t_A) = 0$ (pointing along the x axis) when the field is at an angle $\phi_f(t_A) = \omega_0 t_A$ will, after leaving the first subcoil, be pointing at an angle, $\phi_{(1)} = 2\omega_0 t_A$, as seen from eq. (1) with $\phi(t_A) = t_\pi = 0$.

The field in the second subcoil (reversed B and direction of rotation of B_r) will be at an angle $\phi_f^{(2)} = -\omega_0 t_A$ and when leaving this subcoil the neutron spin will be pointing in a direction (fig. 4) $\phi^{(2)} = -4\omega_0 t_A$, and successive coils will produce precession angles of $6\omega_0 t_A$, $-8\omega_0 t_A$, etc.

If we now consider further replacing the coils B, C, D in the NRSE system (fig. 3b) by a similar set of N subcoils, it can be shown that the effect of t_π cancels and at the exit of the set of D subcoils, the neutron spin will have acquired a precession angle

$$\phi_D = 2N\omega_0(T_{CD} - T_{AB}). \quad (4)$$

In eq. (4) the factor N represents the improvement in performance made possible by the bootstrap in comparison with ordinary NRSE (eq. (3)). Successful operation of such a bootstrap system with $N = 2$ has recently been achieved at Garching [9].

5. Neutron–antineutron oscillations

Following the suggestion by Mohapatra and Marshak [10] that there may exist a baryon number violating interaction ($\Delta B = 2$) capable of transforming neutrons into antineutrons (and vice versa) with the result that neutrons would change into antineutrons after a finite time, there has been a growing interest in the experimental observation of such oscillations in free neutrons [11,12]. The idea of the experiments is to allow a neutron beam to travel over a relatively long distance and then let the beam interact with matter inside a detector where the energy released by the annihilation of any antineutrons present can be detected.

Another possibility, to search for $\bar{n}$'s produced by ultracold neutrons (UCN) [3] stored in a material bottle, while attractive because of the long storage times available [13], suffers from the absorption losses and phase shifts caused by collisions with the confining walls [12].

In the following we show how one can discuss an oscillating n–$\bar{n}$ (two-level) system in terms of the dynamics of a spin-1/2 and then show how the bootstrap idea can be used to partially compensate the effects of the wall collisions.

6. Dynamics of n–$\bar{n}$ oscillations

The n–$\bar{n}$ system, in the presence of an external magnetic field and of the $\Delta B = 2$ n–$\bar{n}$ interaction, can be described by the Hamiltonian:

$$H = \begin{bmatrix} -\mu_0 B_0 & \epsilon \\ \epsilon & \mu_0 B_0 \end{bmatrix}, \quad (5)$$

for n ($\bar{n}$) with magnetic moment parallel (antiparallel) to the applied magnetic field. The $\Delta B = 2$, n–$\bar{n}$ interaction, ϵ, (expected to be $\leq 10^{-22}$ eV) couples states with the same spin, and the other spin state is described by a similar Hamiltonian with the diagonal terms reversed in sign. This eq. (5) is analogous to the Hamiltonian for a spin-1/2 system (with basis states taken as eigenstates of σ_z) in a magnetic field with components B_0 along the z axis and $B_x = \epsilon/\mu_0$ along the x axis (fig. 5). The total

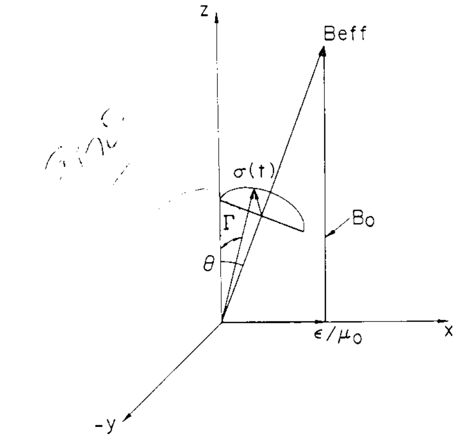

Fig. 5. Effective field for n–n̄ oscillations. B_0 is the applied magnetic field and ϵ is the $\Delta B = 2$ n–n̄ interaction.

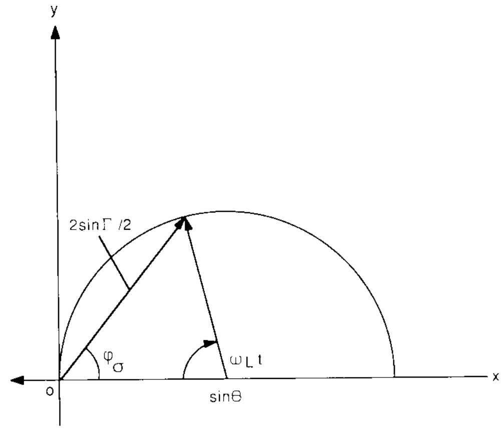

Fig. 6. Motion of the state vector in a plane perpendicular to B_{eff}.

magnetic field makes an angle $\theta = \tan^{-1}\epsilon/\mu B_0$ with the z axis and the amplitude for n̄ can be found by analogy from the amplitude of the $\sigma_z = -1$ state.

Writing the general state of the system as

$$\Psi = \alpha|+\rangle + \beta|-\rangle \Rightarrow \alpha|\mathrm{n}\rangle + \beta|\bar{\mathrm{n}}\rangle, \quad |\alpha|^2 + |\beta|^2 = 1, \tag{6}$$

where $|\pm\rangle$ are the eigenstates of σ_z, we have

$$\langle\sigma_z\rangle = \cos\Gamma = |\alpha|^2 - |\beta|^2, \tag{7}$$

$$\phi_\sigma = \phi_\beta - \phi_\alpha,$$

where Γ and $\phi_\sigma = \tan^{-1}\langle\sigma_y\rangle/\langle\sigma_x\rangle$ are the polar and azimuthal angles of $\langle\boldsymbol{\sigma}\rangle$ and $\phi_{\alpha,\beta}$ are the phases of α, β.

From eqs. (6) and (7) we find the magnitude of β:

$$|\beta| = \sin\Gamma/2 \sim \Gamma/2, \tag{8}$$

where the approximation holds in the case where the system starts off as pure n and $\theta \ll 1$, which holds for any conceivable magnetic field. The state vector precesses around B_{eff} at a rate $\omega_{\mathrm{L}} = 2\mu_0 B_{\text{eff}}$ (fig. 5). Referring to fig. 6, which is the projection on a plane perpendicular to B_{eff}, we see that the system starting as a pure n at $t = 0$ ($\beta(0) = 0$, point 0 in fig. 6) will have a probability of finding an n̄ at t given by

$$P_{\mathrm{n}\bar{\mathrm{n}}}(t) = |\beta|^2 = \sin^2\Gamma/2 = \sin^2\theta\,\sin^2\omega_{\mathrm{L}}/2$$

$$= \frac{\epsilon^2}{\epsilon^2 + (\mu_0 B_0)^2}\sin^2\sqrt{(\mu_0 B_0)^2 + \epsilon^2}\,t. \tag{9}$$

The amplitude of n̄, $|\beta|$, is given by half the distance of the system point from the origin in fig. 6, and the n–n̄ phase difference $\phi_{\mathrm{n}\bar{\mathrm{n}}} = \phi_\sigma$ is also shown in the figure.

As is well known [12,14], eq. (9) shows that $P_{\mathrm{n}\bar{\mathrm{n}}}(t) = (\epsilon t)^2$ for free oscillations ($B_0 = 0$, $\epsilon t \ll 1$) as well as for $\omega_{\mathrm{L}} t \ll 1$ (quasi-free oscillations) which imposes an upper limit on the magnetic field which can be tolerated without loss of sensitivity to the n–n̄ oscillations for a given observation time.

7. Reflection of an ultracold neutron–antineutron (UCN–N̄) system from a wall

To lowest order in ϵ we have $|\alpha| = 1$ in eq. (6). After a wall collision the wave function will become

$$\Psi = \rho_{\mathrm{n}}\,\mathrm{e}^{\mathrm{i}\phi_{\mathrm{n}}}|\mathrm{n}\rangle + \beta\rho_{\bar{\mathrm{n}}}\,\mathrm{e}^{\mathrm{i}\phi_{\bar{\mathrm{n}}}}|\bar{\mathrm{n}}\rangle \tag{10}$$

$$= \mathrm{e}^{\mathrm{i}\phi}\left[\rho_{\mathrm{n}}|\mathrm{n}\rangle + \beta\rho_{\bar{\mathrm{n}}}\,\mathrm{e}^{\mathrm{i}(\phi_{\bar{\mathrm{n}}} - \phi_{\mathrm{n}})}|\bar{\mathrm{n}}\rangle\right] \tag{11}$$

where $\rho_{\mathrm{n}(\bar{\mathrm{n}})}\,\mathrm{e}^{\mathrm{i}\phi_{\mathrm{n}(\bar{\mathrm{n}})}}$ is the reflection coefficient for n (n̄). Since $1 - \rho_{\mathrm{n}} \le 10^{-4}$ for many materials, while $\rho_{\bar{\mathrm{n}}}$ is always smaller, we set $\rho_{\mathrm{n}} = 1$ in eq. (11). Thus we see that the effect of a wall collision at time t will be to rotate the state vector in fig. 7 counterclockwise about the origin by an angle $\mu = \phi_{\bar{\mathrm{n}}} - \phi_{\mathrm{n}}$ and reduce its amplitude by $|\rho_{\bar{\mathrm{n}}}|$.

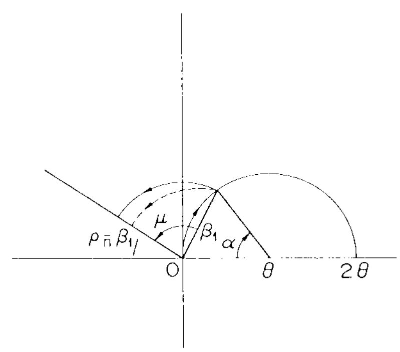

Fig. 7. Wall collision of a UCN–N̄ system. α is the precession angle in the external magnetic field and μ is the phase shift caused by the collision.

We now consider a spherical UCN storage vessel with specularly reflecting walls. We see that successive reflections for a given neutron always occur at the same angle with respect to the surface normal. The distance between collisions is given by $z = 2R\cos\eta$ with η the angle between the trajectory and the radius to the point of collision.

8. The bootstrap applied to the UCN–$\overline{\mathrm{N}}$ system

A particle starting as a pure neutron at $t = 0$ will precess around the point Θ in fig. 7 by an angle α:

$$\alpha = \omega_L t_c = \omega_L z/v = \frac{\omega_L}{v} 2R\cos\eta, \tag{12}$$

in a clockwise direction. At this point a wall collision will rotate the vector by an angle μ, counterclockwise around 0, simultaneously reducing its magnitude by a factor $|\rho_{\bar{n}}|$. As μ is a function of ($v\cos\eta$) only, this sequence:

(a) free motion of the particle represented by a cw rotation around Θ by an angle α, then
(b) a collision with the wall represented by a ccw rotation around 0 by an angle μ and a reduction of radius by $\rho_{\bar{n}}$,

will repeat itself for a given neutron as long as the collisions remain specular.

Letting β_N be the $\bar{n}$ amplitude just before the Nth wall collision, i.e. just before step (b) above, we can show that [2]

$$\beta_N = \frac{\Theta[1 - e^{-i\alpha}][1 - \rho^N e^{iN(\mu-\alpha)}]}{1 - \rho e^{i(\mu-\alpha)}}. \tag{13}$$

For the idealized case $\rho = 1$ we have

$$|\beta_N| = 2\Theta \sin\frac{\alpha}{2} \frac{\sin(\mu-\alpha)N/2}{\sin(\mu-\alpha)/2}, \tag{14}$$

which describes a motion alternatively jumping between two circles and returning to zero after $N = 2\pi/(\mu - \alpha)$ wall collisions. For $\mu = \alpha$, we have a resonance, the phase shift caused by the magnetic field during free flight exactly cancelling the phase shift caused by the wall collisions. For this case eq. (14) yields

$$\beta_N = 2N\Theta \sin\alpha/2, \tag{15}$$

the circular loci of the previous case degenerating into two straight lines making angles $\pm\alpha/2$ with the y axis. There is then a constant growth in $\bar{n}$ amplitude given by

$$\delta\beta = \Theta\alpha = \Theta\omega_L t_c, \tag{16}$$

or just the free space growth rate.

For $\rho < 1$ we see that for $N \to \infty$ the result approaches a constant

$$|\beta_N \to \infty| = \frac{2\theta \sin\alpha/2}{\sqrt{1 + \rho^2 - 2\rho\cos(\mu - \alpha)}}. \tag{17}$$

This means that after an initially curved motion away from the origin the system approaches a steady state where the increase in amplitude due to free oscillations during the time between collisions is exactly compensated by the reduction in amplitude during the collision.

Since the loss probability per wall collision $(1 - \rho_{\bar{n}}^2)$ is due entirely to $\bar{n}$ annihilation in the walls, at each collision the probability P for an annihilation to take place is

$$P = |\beta_N \to \infty|^2 (1 - \rho_{\bar{n}}^2). \tag{18}$$

These annihilations can serve as a means of detecting the n–$\bar{n}$ oscillations. The probability of $\bar{n}$ annihilation during a time interval Δt is

$$P_{\bar{n}}\,\Delta t = P\,\Delta t/t_c. \tag{19}$$

A discussion of the results obtained by integrating over the entire phase space and considering the effects of the Earth's gravitational field has been presented in ref. [2].

9. Discussion of the bootstrap

We have seen that the bootstrap motion – alternate rotations about two different centers resulting in a progressive motion away from the origin – can be useful in two such different systems as the dynamics of the neutron spin as applied to neutron scattering spectroscopy (spin echo) and the search for n–$\bar{n}$ oscillations. The connection between the two systems is the fact that any two-level system is exactly equivalent to a spin-1/2 [15]. In our first case (neutron spin echo) the physical quantity of interest is the direction of the rotating magnetic field at the time a given neutron enters the field. The bootstrap results in the neutron spin pointing at an angle which is a linear amplification of the angle between the rotating field at this instant and the x axis, as seen in fig. 4. This amplification leads directly to an improved performance of the spectrometer. In the n–$\bar{n}$ case, what one wishes to measure is the angle made by the effective magnetic field with the z axis or, more explicitly, we wish to measure the magnitude of the x component of the "magnetic field" in the presence of a much larger z component. In fig. 8 we have seen how the bootstrap can be useful in this case.

What these two applications have in common is that the physical quantity of interest, whose effects one wishes to increases, is a field direction or field component. Other applications where the physically important quantity is a precession of the neutron spin with respect to a field direction, e.g. the search for a neutron edm [5] or parity-violating spin precessions [6], are not improved by the bootstrap.

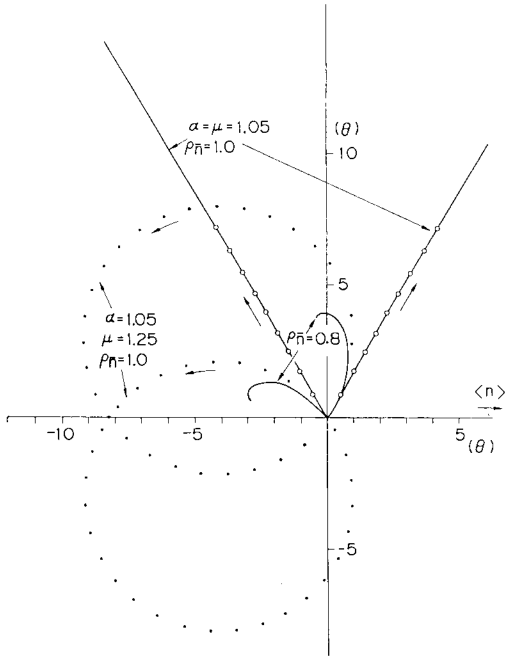

Fig. 8. Buildup of n̄ amplitude for various conditions. For $\rho_{\bar{n}} = 1$, $\mu \neq \alpha$, the locus of the system is two circles, for $\rho_{\bar{n}} = 1$, $\mu = \alpha$, the circles degenerate into straight lines (resonance), for $\rho_{\bar{n}} < 1$, the system approaches an equilibrium point.

The obvious question now is: are there any other cases where the properties of the bootstrap, as discussed above, can be useful?

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