

$$\frac{\hbar \omega_f}{\hbar \omega_i} \sum_{i,j} a_i a_j |\langle n_f | e^{i\vec{Q} \cdot \vec{R}_i(t)} | n_i \rangle|^2 \delta(E_f - E_i)$$

$$\delta(x) = \int e^{ixy} dy$$

$$e^{i\vec{Q} \cdot \vec{R}_i(t)} = e^{i\omega t} e^{i\vec{Q} \cdot \vec{R}_i(0)} e^{-iHt}$$

$$\frac{\hbar \omega_f}{\hbar \omega_i} = \delta(Q, \omega)$$

$$= \frac{\hbar^2}{\epsilon_0} \left(\frac{1}{L^3} \sum_{i,j} a_i a_j \left| \langle n_f | e^{i\mathbf{q} \cdot \mathbf{R}_i} | n_i \rangle \right|^2 \delta(E_f - E_i) \right)$$

$$G(\mathbf{r}, t) = \left(\frac{1}{L^3} \sum_{n_i, n_f} \sum_{i,j} a_i a_j \langle n_i | e^{-i\mathbf{q} \cdot \mathbf{R}_i} | n_f \rangle \langle n_f | e^{i\mathbf{q} \cdot \mathbf{R}_i} | n_i \rangle \delta(E_f - E_i - \hbar\omega) e^{-i\mathbf{q} \cdot \mathbf{r} + i\omega t} d\omega d^3q \right)$$

$$\frac{1}{\hbar} \int d^3q \langle n_i | e^{-i\mathbf{q} \cdot \mathbf{R}_i} | n_f \rangle \langle n_f | e^{i\mathbf{q} \cdot \mathbf{R}_i} | n_i \rangle e^{-i\mathbf{q} \cdot \mathbf{r}} e^{i(E_f - E_i)t/\hbar} e^{i\mathbf{q} \cdot \mathbf{r}} e^{-iE_f t/\hbar} e^{-iE_i t/\hbar} e^{i\mathbf{q} \cdot \mathbf{r}} e^{-iH_f t/\hbar} e^{-iH_i t/\hbar} | n_i \rangle$$

$$G(\mathbf{r}, z) = e^{i\mathbf{q} \cdot \mathbf{R}_i(z)}$$

$$\frac{1}{\hbar} \int d^3q \langle n_i | e^{-i\mathbf{q} \cdot \mathbf{R}_i} | n_f \rangle \langle n_f | e^{i\mathbf{q} \cdot \mathbf{R}_i} | n_i \rangle e^{-i\mathbf{q} \cdot \mathbf{r}}$$

$$\frac{1}{L^3} \sum_{n_i, n_f} \rho_{n_i} \sum_{i,j} a_i a_j \int d^3q \langle n_i | e^{i\mathbf{q} \cdot \mathbf{R}_i(\omega) \cdot \mathbf{R}_i} | n_i \rangle e^{-i\mathbf{q} \cdot \mathbf{r}}$$

$$G_{i,t}$$

$$\frac{1}{L^3} \sum_{n_i} p_{n_i} \sum_{i,j} a_i a_j \int d^3 q \langle n_i | e^{i q \cdot (R_j(z) - R_i)} | n_i \rangle e^{-i q \cdot r}$$

$$\langle a_i a_j \rangle = \langle a \rangle^2 + \delta_{ij} \left(\langle a^2 \rangle - \langle a \rangle^2 \right)$$

$$G_{i,t}$$

$$\frac{1}{L^3} \sum_{n_i} p_{n_i} \sum_{i,j} a_{i,n_i}^2 \int d^3 q \langle n_i | e^{i q \cdot (R_j(z) - R_i)} | n_i \rangle e^{-i q \cdot r}$$

$$G_{self}(r,t) = \frac{1}{L^3} \sum_{n_i} p_{n_i} \sum_i a_{i,n_i}^2 \int d^3 q \langle n_i | e^{i q \cdot (R_i(z) - R_i)} | n_i \rangle e^{-i q \cdot r}$$

$$\langle n_i | \int d^3 q e^{i q \cdot R} e^{-i q \cdot \vec{r}} | n_i \rangle = \frac{1}{(2\pi)^3} \delta^{(3)}(\vec{r} - \vec{R})$$

$$G_{i,t} = \int d^3 r' \delta^3(r + R_i - r') \delta^3(r' - R_j(z)) | n_i \rangle = \langle n_i | \delta^3(r + R_i - R_j(z)) | n_i \rangle$$

$$\frac{d^2\sigma}{d\Omega d\omega} = \frac{\hbar_c}{\hbar_i} S(\Omega, \omega) (a^2)_{inc} \int dz dr G_{self}^{sys}(r, z) e^{i\vec{\omega} \cdot \vec{r} - \omega z}$$

$$E_i > \omega = a^2 \left(S(\Omega, \omega) \int dr G(r, z) e^{i\vec{\omega} \cdot \vec{r}} \right) \left(1 + \int dr g(r) e^{i\omega r} \right) = S(\omega)$$