$$\sum_i \frac{a_i}{|\vec r - \vec R_i|} e^{ik|\vec r - \vec R_i|} e^{i\vec k_1 \cdot \vec R_i}$$

$$r \gg R_i$$

$$|\vec r - \vec R_i| = \sqrt{(\vec r - \vec R_i)^2}$$

$$= \left(r^2 - 2\vec r\cdot\vec R_i + R_i^2\right)^{1/2}$$

$$= \left(r - \frac{\vec r}{r}\cdot \vec R_i\right) k$$

$$e^{ikr} \; e^{-i\vec k_2 \cdot \vec R_i}$$

$$k_2 = (\hat r k)$$

$$\left(\frac{a_i}{r} e^{ikr}\right) \quad e^{i(\vec k_1 - \vec k_2)\cdot R_i}$$

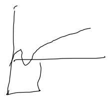
$$e^{iQ\cdot R_i}$$

$$Q = \vec k_1 - \vec k_2$$

$$\langle k_f | V \sim (a\,\delta(r - R_i)) | k_i \rangle$$

$$\left[\frac{e^{ikr}}{r}\right] \sum_i a_i \cancel{\frac{e^{ikr}}{r}} e^{i\vec Q\cdot\vec R_i}$$

$$-\frac{\hbar^2}{2\mu} \nabla^2\psi + V\psi = E\psi$$



$$V_F(r) = \sum_i \frac{2\pi\hbar^2}{\mu} a_i \,\delta(r - R_i)$$

$$\frac{a_B}{m} = \left(\frac{a}{\mu}\right)$$

$$\frac{d\sigma}{d\Omega} = \left| \sum_i a_i e^{i\vec Q\cdot\vec R_i} \right|^2$$

$$= \left\langle \sum_{ij} (a_i^* a_j) e^{i\vec Q\cdot(\vec R_i - \vec R_j)} \right\rangle$$

$$\sum_{ij} \langle a_i a_j^* \rangle e^{iQ\cdot(\vec R_i - \vec R_j)}$$

$$i \neq j \quad \langle a_i a_j^* \rangle = \langle a_i \rangle \langle a_j \rangle$$

$$= \langle a \rangle^2$$

$$i = j \quad \langle |a|^2 \rangle$$

$$\langle a_i a_j^* \rangle = \langle a \rangle^2 + \delta_{ij} \underbrace{\left[\langle |a|^2 \rangle - (\langle a \rangle)^2\right]}_{\text{ms dev.}} \quad [A^2]$$

$$\sum_{ij} \underbrace{\langle a \rangle^2 e^{iQ(R_i - R_j)}}_{\text{coherent}} + A_{inc}^2 N$$

Multiple scattering

Sears "Neutron Optics"

$\langle a_i a_j^* \rangle = \langle a \rangle^2 + \delta_{ij}\,[\underline{\langle |a|^2 \rangle - (\langle a \rangle)^2}]$

ms dev.

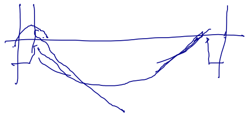
$\sum_{ij} \langle a \rangle^2 e^{iQ(R_i - R_j)} + A^2_{inc} N$

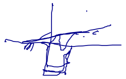
coherent

Multiple scattering

Sears "Neutron Optics"

$$\psi(r) = e^{i k_i \cdot r} + \sum_j C_j \frac{e^{ik|r-R_j|}}{|r-R_j|}$$





$$\psi(r \to R_i) = C_i \left[\frac{1}{|r-R_i|} - \frac{1}{a_i}\right] =$$

$$e^{i k_i R_i} + \frac{C_i}{\cancel{|r-R_i|}}\left(\frac{1}{|r-R_i|} + ik\right) + \sum_{j \neq i} C_j \frac{e^{ik(R_i-R_j)}}{R_i - R_j}$$

$-e^{i k_i \cdot R_i}$ $0 = C_i\left(\frac{1}{a_i} + ik\right) + \sum_{j \neq i} C\,)$

$= \sum_j \Gamma(k)_{ij} C_j$ $\quad \Gamma_{ii} = \left(\frac{1}{a_i} + ik\right)$

$\Gamma_{ij} = \frac{e^{ik(R_i - R_j)}}{|R_i - R_j|}$

$$-C_j = \sum_i [\Gamma(k)]^{-1}_{ij} e^{i k_0 \cdot R_i}$$

$$\psi(r \to \infty) = e^{i k_i \cdot r} - \frac{e^{ikr}}{r} \sum_{ij} \Gamma(k)^{-1}_{ij} e^{i k_i R_i - i k_f R_j}$$

$$\frac{e^{ikr}}{r} \sum_{ij} [\Gamma(k)]^{-1}_{ij} e^{i k_0 \cdot R_i} e^{-i k_s \cdot R_j}$$

Forward Scatter $\quad k = 0$

$a_{eff} = \sum_{ij} [\Gamma(0)]^{-1}_{ij}$

SPIN INCOHERENCE

$$a = a_0 + \vec{S}\cdot\vec{I}\, a_s = a_+ 1_+ + a_- 1_-$$

S = neutron, I = nucleus (spin)

$$J_+ = I + \tfrac{1}{2} \qquad |J_-\rangle = I - \tfrac{1}{2}$$

$$\vec{J} = \vec{I} + \vec{S} \qquad J^2 = I^2 + S^2 + 2\,\vec{I}\cdot\vec{S}$$

$$J(J+1) = I(I+1) + S(S+1) + 2\,\vec{I}\cdot\vec{S}$$

$$(a_+ - a_-) = a_s\left((S\cdot I)_+ - (S\cdot I)_-\right)$$

$$\left[(I+\tfrac{1}{2})(I+\tfrac{3}{2}) - (I-\tfrac{1}{2})(I+\tfrac{1}{2})\right]\tfrac{1}{2}$$

$$\frac{(2I+1)}{2} = (I+\tfrac{1}{2})$$

$$a_+ - a_- = a_s\,[I+\tfrac{1}{2}] \qquad \left(a_s = \frac{a_+ - a_-}{I+\tfrac{1}{2}}\right)$$

$$a_+ = a_0 + a_s (S\cdot I)_+$$

$$a_0 = a_+ - \frac{(a_+ - a_-)}{2(I+\tfrac{1}{2})}\left[\cancel{(I+\tfrac{1}{2})(I+\tfrac{3}{2}) - I(I+1) - S(S+1)}\right]$$

$$I$$

$$= \left(\frac{a_+(I+1) + a_- I}{2I+1}\right)$$