Spin Dynamics

$S = \frac{1}{2}\hbar$ $\qquad \vec{S}\cdot\vec{S} = s(s+1)\hbar^2$

$\frac{3}{4}$

$[L_x, L_y] \sim L_z$ $\qquad \vec{S} = S_x \hat{x} + S_y \hat{y} + S_z \hat{z}$

$$\psi = a\psi_+ + b\psi_-$$

$$= \begin{pmatrix} a \\ b \end{pmatrix} \qquad \frac{\vec{\sigma}}{2} = \vec{S}$$

$|\psi\rangle = a|+\rangle + b|-\rangle$

Pauli

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$[\sigma_x, \sigma_y] = 2i(\sigma_z)$$

$$\underline{\sigma_x\sigma_y + \sigma_y\sigma_x = 0}$$

$$\sigma_i^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad (\vec{\sigma}\cdot\vec{A})^2 = (A)^2 \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$i\hbar \frac{\partial \psi}{\partial t} = H\psi \qquad \psi_t = e^{-i\cancel{H}t/\hbar}\psi_0$$

$$= e^{-\frac{i}{\hbar}\int H\,dt}\,\psi_0$$

$$H(t_1)\,\Delta t + H(t_2)\,\Delta t \ldots$$

$$e^{A+B} \neq e^{AB} \qquad [A,B]=0$$

$$\omega_L = \gamma B_0 \qquad H = -\vec{\mu}\cdot\vec{B}(t)$$

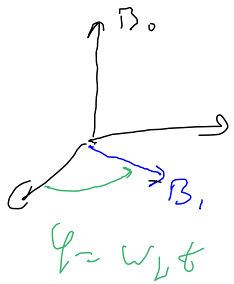
$$\vec{\mu} = \frac{\gamma}{2}\vec{\sigma} \qquad \frac{\gamma}{2}\vec{\sigma}\cdot\vec{B}(t)$$

$$\sigma_z\, B(t)$$

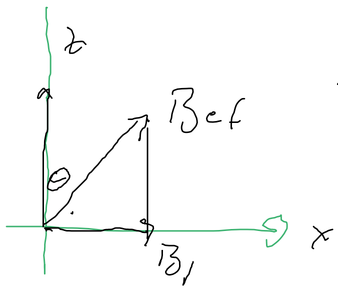
$$B \neq f(t)$$

$$\psi = \left[e^{+\frac{i}{\hbar}\frac{\gamma}{2}\vec{\sigma}\cdot\vec{B}t}\right]\psi_0$$

$$R = \begin{pmatrix} \rho_+ & 0 \\ 0 & \rho_- \end{pmatrix}$$



$\begin{pmatrix} B_0 - \frac{\omega}{2\gamma} \\ 0 \end{pmatrix}$



$\sin\theta = \frac{B_1}{B_{eff}}$

$B_{eff} = \sqrt{(B_0 - \frac{\omega}{\gamma})^2 + B_1^2}$

$e^{i\frac{\gamma}{2}\vec{\sigma}\cdot\vec{B}_{eff} t}$

$\psi_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cos\left(\frac{\gamma}{2} B t\right) + i \frac{\sin(\frac{\gamma}{2}Bt)}{\frac{\gamma}{2}Bt} \times \left\{ \right.$

$\sigma_x B_x + \sigma_z B_z$

$\left\{ \begin{bmatrix} B_0 - \omega/\gamma & B_1 \\ B_1 & -(B_0 - \omega/\gamma) \end{bmatrix} \left(\frac{\gamma}{2} t\right) \right.$

$\psi(t) = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$\psi_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}_z$

$\psi(t) = U(t)\,\psi_0$

$P_{+\to-} = |C|^2$

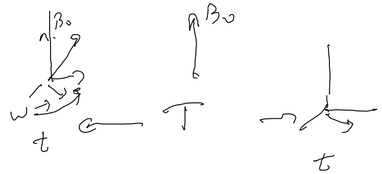
$C_{+-} = \frac{i \sin \frac{\gamma}{2} B t}{\gamma/_2 B t} \; B_1 \frac{\gamma}{2} t$

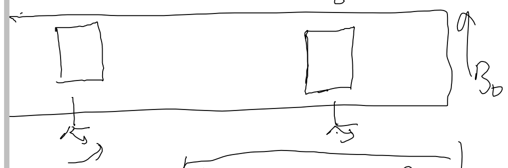
$|C|^2 = \left(\frac{B_1}{B}\right)^2 \sin^2 \frac{\gamma}{2} B t$

$\left[\frac{B_1^2}{(B_0 - \frac{\omega}{2\gamma})^2 + B_1^2}\right] \sin^2 \frac{\gamma}{2}\left(\sqrt{(B_0^2 - \omega/_{2\gamma})^2 + B_1^2}\right) t$

$\sin^2 \gamma B_1 t$

Ramsey Method - Separated Oscillating fields Method





$$\gamma B = \left(\sqrt{\left(B_0 - \frac{\omega}{2\gamma}\right)^2 + B_1^2}\right)\gamma = a$$

$$\frac{B_1}{B} = \sin\theta$$

$$\frac{\left(B_0 - \frac{\omega}{2\gamma}\right)}{B} = \cos\theta$$

$$\lambda = \left(B_0 - \omega/2\gamma\right)\gamma$$

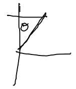
rotating fields

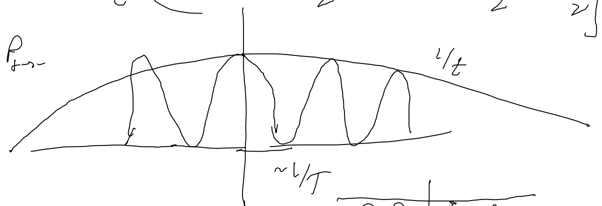
$$P = e^{i\frac{\gamma}{2}\sigma\cdot\vec{B}_{eff}} = \begin{pmatrix} \cos\frac{at}{2} + i\cos\theta\sin\frac{at}{2} & i\sin\theta\sin\frac{at}{2} \\ & \cos\frac{at}{2} - i\cos\theta\sin\frac{at}{2} \end{pmatrix}$$

$$F = \begin{pmatrix} \cos\frac{\lambda T}{2} + i\sin\frac{\lambda T}{2} & 0 \\ 0 & \cos\frac{\lambda T}{2} - i\sin\frac{\lambda T}{2} \end{pmatrix}$$

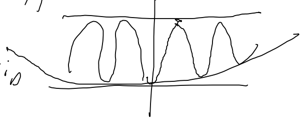
$$U = PFP$$

$$C_{+\to-} = 2i\sin\theta\left\{\frac{1}{2}\sin at\cos\frac{\lambda T}{2} + \cos\theta\sin\frac{\lambda T}{2}\sin^2\frac{at}{2}\right\}$$

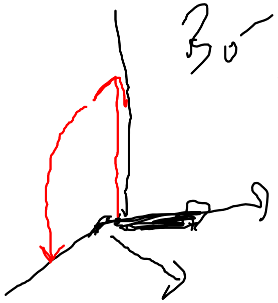




$1 - P_{+\to-} = P_{noflip}$



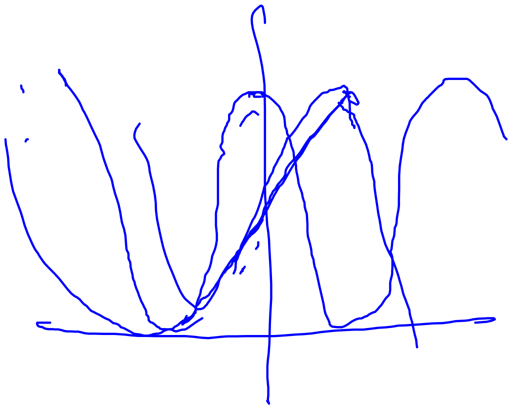
$B_0 - \frac{\omega}{\gamma} = 0$ on r...



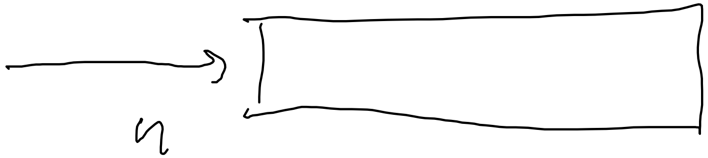
$$\left[(\omega_L - \omega) T \right] = 8\pi$$

$$(\omega_L - \omega) T + \varphi = 0$$

φ = phase ~~shift~~ shift between coils



$$B_{eff} \sim B \vec{I} \rho_t$$



$$f = A + \rho_t (\vec{\sigma} \cdot \vec{I})^B + C \vec{\sigma} \cdot \vec{k}$$

$$+ \rho_t D (\vec{\sigma} \cdot \vec{I} \times \vec{k})$$

$$V = \left(\frac{2\pi\hbar^2}{m} N f \right)$$

$$\left(U(t) = e^{-iVt/\hbar} \right)$$

2x2 matrix

$$G = \left(a + \vec{\sigma} \cdot \vec{b} \right)$$

$$G_{11} = a + b_z$$

$$G_{22} = a - b_z$$

$$G_{\substack{12 \\ 21}} = b_x \pm i b_y$$