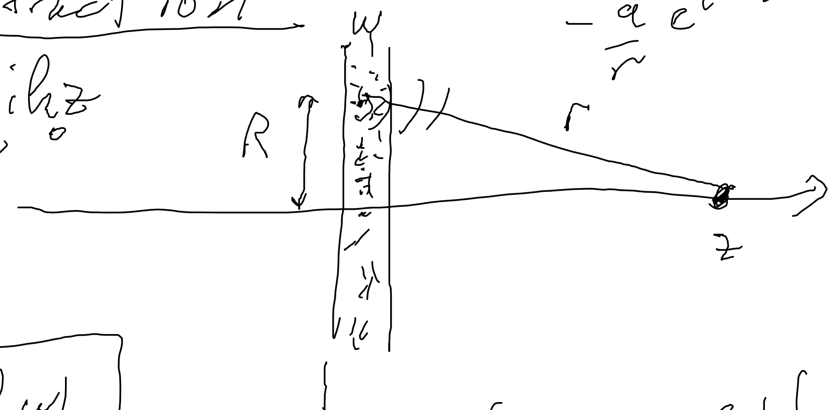
Ultra-Cold Neutrons

index of refraction

$c' = c_0/n$ $\quad e^{ik_0 z}$ $\quad -\frac{a}{r} e^{ikr}$



$z=0$

$k' = n k_0$

$$\boxed{e^{ik_0(z-w)}\, e^{i(n k_0 w)}}$$

$$-a\, 2\pi\, w\rho \int \frac{e^{ikr}}{r} R\,dR$$

$$= e^{ik_0 z}\left[e^{i(n-1)k_0 w}\right] \qquad R\,dR = r\,dr \qquad \frac{-a\, 2\pi\, w\rho}{ik_0}\left(\int_z^\infty e^{ik_0 r}\,dr\right)$$

$$r^2 = R^2 + z^2 \qquad = e^{ik_0 z} \qquad + \frac{2\pi a w\rho\,(1+e^{ik_0 z})}{ik_0}$$

$$= e^{ik_0 z}\left[1 \mp i\, 2\pi a w\rho / k_0\right]$$

$$= e^{ik_0 z}\left[e^{i(n-1)k_0 w}\right] = \left[1 + i(n-1)k_0 w\right] e^{ik_0 z}$$

$$\psi(r) = \psi_{inc} - \sum_i a_i \frac{e^{ik_0|r-r_i|}}{|r-r_i|} \psi_i(r_i)$$

$e^{ik_0 \cdot r}$

$$\psi_i(r_i) = \psi(r_i)$$

$$= e^{i\vec{k}_0\cdot\vec{r}} - \int [\rho(r_i)a_i]_{r'} d^3r' \, \psi(r') \underbrace{\left[\frac{e^{ik_0|r-r'|}}{|r-r'|}\right]}_{G(r-r')}$$

$$[(\nabla^2 + k_0^2)\psi(r)]$$

$$(\nabla^2 + k_0^2) G(r-r') = -4\pi\delta^3(r-r')$$

$$= 4\pi[\rho a]_r \psi(r) \Big] \left(-\frac{\hbar^2}{2m}\right)$$

$$\left(-\frac{\hbar^2}{2m}\nabla^2 - \cancel{\frac{\hbar^2}{2m}k_0^2}\right)\psi + 4\pi[\rho a]_r \frac{\hbar^2}{2m}\psi(r) = 0$$

$$= \frac{\hbar^2 k_0^2}{2m}\psi_0$$

$$-\frac{\hbar^2}{2m}\nabla^2\psi + \frac{\hbar^2}{m}2\pi[\rho a]_r \psi(r) = E_0\psi_0$$

$$V(r) = \frac{2\pi}{m}\hbar^2[\rho a]_r$$

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = E\psi$$

$$\left(\frac{V-E}{E}\right)$$

$$\frac{\hbar^2 k_0^2}{2m}\psi + (V-E)\psi$$

$$\nabla^2\vec{E} + n k_0^2 \vec{E} = 0$$

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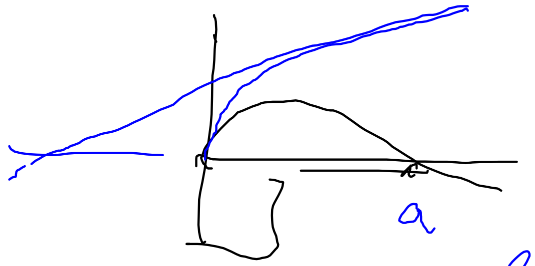
$$= e^{ik_0 t}\left[1 \cancel{\mp} i\,2\pi a w \delta / k_0\right]$$

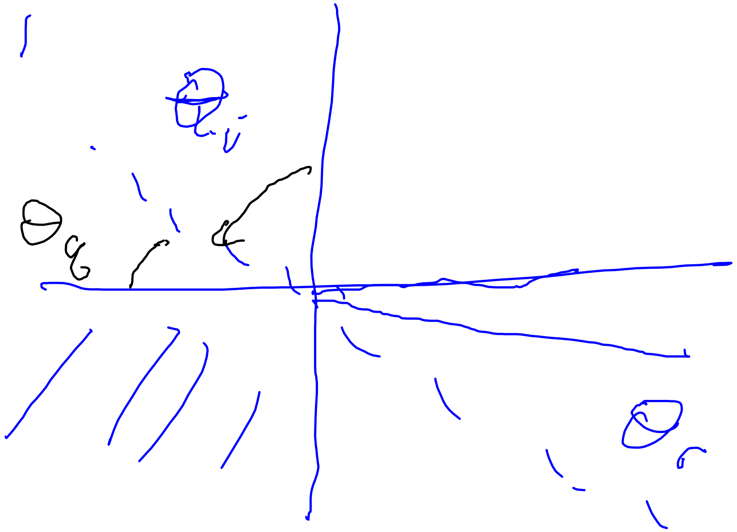
$$= \left[1 + i(n-1)k_0 w\right]$$



$$\therefore (1-n) = \left(\frac{2\pi a \delta}{k_0^2}\right) = \left(\frac{2\pi N a}{k_0^2}\right) \quad \therefore \quad n = 1 - \frac{2\pi N a}{k_0^2}$$

$n < 1$

$$\frac{\sin\theta_i}{\sin\theta_r} = \frac{n_r}{n_i} = n_r$$



$$\sin\theta_c = n_r = \cos\theta_g$$

$$\boxed{(n^2-1) \to (n-1)\overset{2}{(n+1)} = 4\pi N a / k_0^2}$$

$$\nabla^2 E - \frac{n^2}{c^2}\frac{\partial E}{\partial t^2} = 0$$

$$-k^2 E + \frac{n^2}{c^2} \omega^2 E = 0$$

$$k^2 = \left(\frac{n^2 \omega^2}{c^2}\right)$$

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = E\psi$$

$$\frac{\hbar^2}{2m}\nabla^2\psi + (E-V)\psi$$

$$k^2 \Longrightarrow \frac{2m}{\hbar^2}(E-V)$$

$$-\frac{\hbar^2 k^2}{2m}\psi + (E-V)\psi = 0 \quad 2m/\hbar^2$$

$$n = (k/k_0)$$

$$n^2 = \frac{k^2}{k_0^2} = \frac{2m}{\hbar \, k_0^2}(E-V) = \left(\frac{E-V}{E}\right)$$

$$n^2 = 1 - \frac{V}{E} \Rightarrow \quad n^2 - 1 = -\frac{V}{E}$$

$$(n-1)(n+1) = -\frac{V}{2E}$$

$$n - 1 = \frac{2\pi}{m}\frac{\hbar^2 [\beta a]}{E\,\hbar^2 k_0^2}$$

$$\frac{2\pi}{m}[\beta a]_r \psi(r) = \psi_0$$

$$V(r) = \frac{2\pi}{m}\hbar^2 [\beta a]_r$$

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