$$\vec{u}_i = \sum_{s,q} \sqrt{\frac{\hbar}{2\omega_{s,q} M N}}\ \vec{\gamma}(s,q)\left[a_{s,q} + a^{\dagger}_{s,q}\right]$$

$$= \left(\xi_i(s,q)\, a_{s,q} + \xi^{*}(s,q)\, a^{\dagger}_{s,q}\right)$$

$$\langle n_f | e^{iQ\cdot(\rho+u)} | n_0 \rangle$$

$$\prod_{s,q} \langle n_f | e^{iQ\cdot\rho_i} \left(e^{iQ\cdot(\xi a + \xi^{*} a^{\dagger})}\right) | n_0 \rangle$$

$$1 + i(Q\cdot\xi a + Q\cdot\xi^{*} a^{\dagger}) - (Q\cdot\xi)^2 a^2 + (a^{\dagger})^2 + (a a^{\dagger})\xi^2 + (a^{\dagger} a)$$

$$\left(1 - \frac{Q^2\xi^2}{2}(2n+1)\right)$$

$$e^{iQ\cdot\rho_0} \prod_{s,q} \left[1 - \frac{(Q\cdot\xi)^2}{2}(2n+1)\right]$$

$$e^{-W(Q)}$$

$$W(Q) = \sum_{s,q} \left(Q\cdot\xi_{s,q}\right)^2 \left(2n_{s,q}+1\right)$$

$$\sum_{s,q} \frac{1}{2}\, \frac{|Q\cdot\vec{\gamma}|^2 \hbar}{2M\omega_s(q) N}\left(2n_{s,q}+1\right) \qquad \langle n \rangle = \frac{1}{\left(e^{\hbar\omega/k T} - 1\right)}$$

$$d\omega\, g(\omega) = \frac{1}{3N} \sum_{s,q} \delta(\omega - \omega_{s,q})\, d\omega$$

$$\int g(\omega)\, d\omega = 1$$

Debye Waller

$$\boxed{\frac{3\hbar}{4M} \int d\omega\, \frac{g(\omega)}{\omega} |Q\cdot\gamma|^2 \coth\left(\hbar\omega/2k_B T\right)}$$

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$$\langle n_f | e^{iQ\cdot R_i} | n_i \rangle = \beta_i = \langle f | e^{iQ\cdot \rho_i} e^{iQ\cdot u_i} | i \rangle$$

$$R_i = \rho_i + u_i \qquad u_i \sim \vec{\gamma}_{s q} \, e^{i(\vec{q}\cdot \vec{\rho}_i - \omega_{sq} t)}$$

$$\beta_i = e^{i(Q \pm q)\cdot \rho_i} e^{-\omega(q)} \; \underline{i(\vec{Q}\cdot\vec{\gamma})} \qquad \xi a + \xi^+ a^+$$

$$\left(\frac{\hbar}{2mN\hbar\omega_{sq}}\right)^{1/2} \qquad \sqrt{n_i} \qquad \sqrt{n_i+1}$$

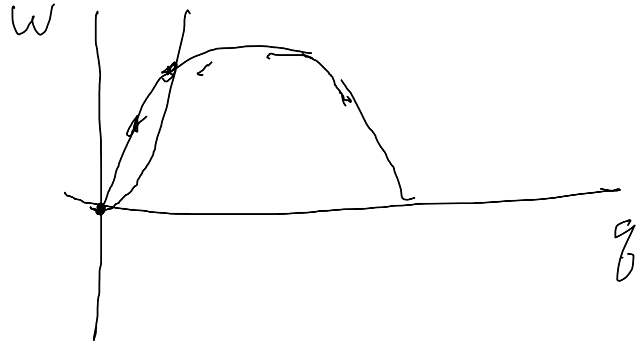
$$1 + i Q\cdot(\xi a + \xi^+ a^+) - \frac{Q^2}{2} \xi^2 (\quad)$$

$$\left|\sum_i \beta_i\right|^2 \to \left|\sum_i e^{i(Q \pm q)\rho_i}\right|^2$$

$$\Rightarrow \underline{\delta(\vec{Q} \pm \vec{q} + \vec{\tau})}$$

$$\frac{d^2\sigma_{coh}}{d\omega\, d\Omega} \sim \left|\sum_i \beta_i\right|^2 \underline{\delta(E)}$$

$$\hbar\omega - (E_f - E_i)$$



$$w = \frac{p_f^2}{2m} = \frac{\hbar^2 Q^2}{2m}$$

β_i $\quad \sum_i (\beta_i)^2 = \hbar\, e^{-2W} \, |Q \cdot \gamma|^2 \cdot \begin{pmatrix} n \\ n+1 \end{pmatrix} \begin{matrix} a \\ a^+ \end{matrix}$

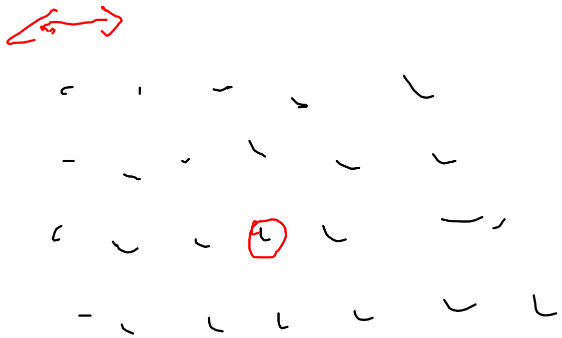
$$\sum_{s,q} \frac{}{2M N w_s(q)}$$

$$\frac{d^2\sigma_{inc}}{dw\, d\Omega} = a^2 \frac{k_f}{k_i} \delta(E) \qquad \frac{\langle n \rangle}{w} = \left(\frac{1}{e^{\hbar w/T} - 1}\right)_w = \psi(w)$$

$$\frac{\langle n+1 \rangle}{w} = \frac{1 + e^{w/T} - 1}{e^{w/T} - 1} = \left(\frac{1}{1 - e^{-w/T}}\right)_w$$

$$a_{inc}^2 \frac{k_f}{k_i} |Q \cdot r|_{ave} \quad \frac{1}{3}(Q^2) \quad \boxed{\int g(w)\, dw\, \psi_{\pm}(w)\, e^{-2W} \frac{3\hbar}{2M}}$$

incoherent approximation



$$\left|\sum_i e^{i(Q\pm q)\cdot\vec{r}_i}\right|$$

$$r_i = \sum \vec{a}_i \, x_i$$

$$(Q\pm q)\cdot\vec{a}_i = 2\pi n$$

$$(Q\pm q) = -\vec{\tau}$$

$$a\cdot\tau = 2\pi n$$

$$\langle n_f | iQ\cdot\xi\,(a + a^\dagger) | n_i \rangle$$
$$\xi^*$$

$$i(Q\cdot\xi)\underline{\langle n_f | a | n_i \rangle} + \langle n_f | a^\dagger | n_i \rangle\, Q\cdot\xi^*$$

$n_f = n_i - 1$ $\qquad\qquad$ $n_f = n_i + 1$

$\sqrt{n_i}$ $\qquad\qquad$ $\sqrt{n_i + 1}$

either $\qquad\qquad$ OR

Phonons

$$V(r) = \sum_i \delta(\vec{r} - \vec{R}_i)\, b_i$$

$$\langle \quad \rangle \to \langle n_f | e^{iQ \cdot R_i} | n_i \rangle$$

$$R_i = \xi_i + \vec{u}_i(t)$$

$$u_i = a e^{iq \cdot r} + a^+ e^{-iq \cdot r}$$

$$1 + i\vec{Q} \cdot \vec{u}_i - \frac{(Q \cdot u_i)^2}{2}$$

$(10^{-2}\ eV)$

E - M → photons

$$V(r) = \frac{e}{c} \vec{p} \cdot \vec{A}(\vec{r})$$

$$\langle \quad | p \cdot A(r) | \quad \rangle$$

$$A(r) = a_k e^{i \vec{k} \cdot \vec{r}} + a_k^+ e^{-ik \cdot r}$$

$$a^+|n\rangle = b_n |n+1\rangle$$

$$= \sqrt{n+1}\, |n+1\rangle$$

$$\langle n' | a e^{i \vec{k} \cdot \vec{r}} | n \rangle$$

$$\left(1 + \frac{i k \cdot r}{e/m} - \frac{(k \cdot r)^2}{2}\right)$$

$$\left| \sum^N e^{i(\vec{Q} + q) \cdot \vec{\xi}_i} \right|^2 \Rightarrow \delta(\vec{Q} + \vec{q} - \vec{\tau}) \quad \text{coh.}$$

$$\sum \left| e^{i(\vec{Q} + \vec{q}_i) \cdot \vec{\xi}_0} \right|^2 \to \pm f(\vec{Q}, \vec{q}) \quad \text{incoherent}$$