PY 810 Homework 2

1. Polyethylene has the formula $(CH_2)_n$ and has a density of 0.92 g/cc. For C we have

$a_{coh} = .665 \times 10^{-12}$ cm and H has $a_{coh} = -.374 \times 10^{-12} cm$. Take the UCN loss cross-section for Hydrogen as 5 barns/proton referred to thermal velocity (2200 m/sec) (remember loss cross sections vary as $1/v$) and neglect any losses associated with Carbon. Calculate the effective potential for this material and the UCN reflectivity as a function of energy up to 200 nev. You can use the formula

$$V = 157 \frac{\rho\,(g/cc)}{A} b_{coh}(fermis)...nev$$

where $\rho/A = N/(6 \times 10^{23})$..$N$ is the number of nuclei/cm^3 if you derive it once and for all. Also

$$U = 3.3 \times 10^{-7} \cdot N(10^{22}/cm^3) \cdot \sigma\,(barns) \cdot v\,(m/\sec)\,...nev$$

is the imaginary part of the potential.

2. Try to design a material with zero effective potential. I.e. find a reasonable material with a negative scattering length and combine it with something with a positive scattering length with the relative composition chosen to give V=0. What is the imaginary potential for this material? What is the reflection as a function of normal UCN energy for this material assumed to be of infinite thickness?

3. Compare the reflection, as a function of normal energy for energies above the barrier, from a material whose boundary is represented by a step function (V_o=94 nev) with one whose boundary is given by

$$V = \frac{V_o}{1 + e^{-x/a}} \qquad (1)$$

with $a = 73$Å. There is an analytic solution that gives the reflection probability for (1) as

$$|R|^2 = \left(\frac{\sinh \pi (k - k_1) a}{\sinh \pi (k + k_1) a} \right)^2$$

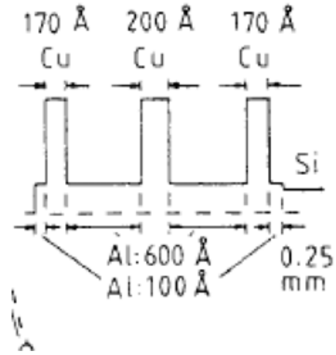
where $k \sim \sqrt{E}$ pertains in the region $x \to -\infty \quad (V = 0)$ and $k_1 \sim \sqrt{E - V_o}$ in $x \to \infty \quad (V = V_o)$.

Calculate the reflection from the potential (1) using one of the multi-layer matrix techniques taking the height of the layers to mimic the behavior (1). How many layers are necessary to give reasonable accuracy.

OPTIONAL for those who received less than 40/50 on Homework 1, required for others

Calculate the reflection probability by numerical integration of the Schroedinger equation. I think the best way to do this is to start at some large x where $V = V_o$ and assign values to ψ and ψ' of $e^{ik_1 x}$ and $ike^{ik_1 x}$.Then integrate back to $x \ll 0$, and find an oscillating wave function. $|\psi|^2$ for this wave function will have an average value given by a normalization factor $|A|^2$, associated with taking the initial transmitted wave function to have amplitude 1, and maxima and minima given by $|A|^2 \, |1+|R||^2$ and $|A|^2 \, |1-|R||^2$. The transmission probability will be given by $1/|A|^2$.

4. a)



Calculate the reflection of UCN as a function of normal energy for a potential based on a simplified version of the figure, i.e. set the potential for Al and Si to be zero. Thus we have three films of copper of thickness 170,200,170 Å separated by 600Å of V=0 material.

b) The center film is contaminated with 10 atomic% of Hydrogen. Neglecting the effect of the real part of the Hydrogen potential calculate the absorption as a function of normal energy using the perturbation method given in class. Use 5 barns/proton referred to thermal velocity for the loss cross section.

c) Calculate the reflection curve taking into account the real as well as the imaginary part of the Hydrogen potential

5. Using equation A3.36 in phonons.pdf on the web site calculate the temperature dependence of UCN upscattering when $g(\omega)$ is given by a Debye spectrum:

$$\begin{aligned} g(\omega) &= \tfrac{3}{\theta^3}\omega^2 \quad \{\omega \leq \theta \\ &= 0 \qquad\quad\; \{\omega > \theta \end{aligned}$$

where θ is the Debye temperature. Take $\left|\overrightarrow{Q} \cdot \overrightarrow{\gamma}\right|^2_{ave} = \frac{Q^2}{3}$

Calculate

$$\frac{k_i}{a_{inc}^2} \int \left(\frac{d^2\sigma}{d\Omega d\omega} \right) d\Omega d\omega \tag{2}$$

using $d\Omega = 2\pi \sin\theta d\theta = \frac{2\pi Q dQ}{k_i k_f}$ and $Q = k_f, \quad dQ = 2k_i$, and then take $k_i = k_{UCN} = 0$. Note that (2) is proportional to the inverse UCN lifetime in the material:

$$\frac{1}{\tau} = N\sigma_{tot} v$$