Production of UCN by downscattering in superfluid He[4]

E. Korobkina [a], R. Golub [a,*], B.W. Wehring [b], A.R. Young [c]

[a] *Hahn Meitner Institut, Glienickerst. 100, 14109 Berlin, Germany*
[b] *Nuclear Engineering Department, North Carolina State University, Raleigh, NC, USA*
[c] *Physics Department, North Carolina State University, Raleigh, NC, USA*

Abstract

Ultra-Cold Neutrons (UCN) are neutrons with energies so low they can be stored in material bottles and magnetic traps. They have been used to provide the currently most accurate experiments on the neutron life time and electric dipole moment. UCN can be produced in superfluid Helium at significantly higher densities than by other methods. The predominant production process is usually by one phonon emission which can only occur at a single incident neutron energy because of momentum and energy conservation. However UCN can also be produced by multiphonon processes. It is the purpose of this work to examine this multiphonon production of UCN. We look at several different incident neutron spectra, including cases where the multiphonon production is significant, and see how the relative importance of multiphonon production is influenced by the incident spectrum. © 2002 Published by Elsevier Science B.V.

1. Introduction

The production of Ultra-Cold Neutrons (UCN) [1] in superfluid Helium and their subsequent storage allowing the build up of densities much greater than would be achievable at thermal equilibrium with the Helium (superthermal UCN source) [2] is the foundation for several current and proposed experiments with UCN [3–6,18]. Although calculations of the production rate have been presented previously [2,7,14] the early calculations concentrated on the production due to the resonant (8.9 Å) phonons. Measurements [8] have shown a rather narrow peak in the production cross section as expected. In the course of the first calculation [2] estimates had been made of the contribution due to higher energy incident neutrons downscattering on the multiphonon excitation tail in $S(q, \omega)$, and these neutrons were found to contribute about the same amount of UCN production as the resonant neutrons for a reasonable assumed spectrum. However as the initial work was part of a proposal for an experiment

it was felt that it would be more conservative not to include this in the initial estimates of UCN density, i.e., to keep this in hand as a factor of safety for the predicted intensity for the new source. However the fact that multiphonon processes would increase the UCN production rate was mentioned.

In recent years the comparison between theory and experiment has improved significantly and there are several proposals to operate a superthermal Helium UCN source with a monochromatic beam of 8.9 Å neutrons, so the question of the production rate due to multiphonon processes has some practical importance. In addition, due to a vigorous experimental program [9], much more detailed information concerning $S(q, \omega)$ has become available. For these reasons it has become necessary to re-examine the situation. Schott, in particular, has emphasized the importance of the UCN production due to higher energy neutrons.

This work is concerned with calculations of the UCN production rate when a volume of superfluid He^4 at temperatures of $\lesssim 0.5$ K is exposed to a neutron flux. The steady state UCN density in the source is given by

$$\rho_{\text{UCN}} = P\tau, \tag{1}$$

where P is the production rate, UCN cm^{-2} s^{-1} and τ is the UCN lifetime against all losses in the Helium filled storage chamber. Lifetimes $\gtrsim 500$ seconds should be achievable. The storage time in superfluid Helium has been measured in [3,11,12].

2. Production of UCN

2.1. Introduction

A neutron at rest can absorb energy $\hbar\omega$ and momentum $\hbar q$ with

$$\omega = \hbar q^2 / 2m \equiv \frac{\alpha}{2} q^2 \tag{2}$$

with ($\alpha = 4.14$ meV Å^{-2}) and contrarily a neutron with this energy and momentum can come to rest after transferring its energy and momentum to the superfluid. For single phonon interactions, which are usually dominant, the superfluid can only exchange quantities of energy and momentum that are related by the dispersion curve

$$\omega = \omega(q) = cq, \tag{3}$$

where the last relation is an approximation to simplify the discussion. Then neutrons can only come to rest by emission of a single phonon if they have the resonant energy E^* given by the intersection of Eqs. (2) and (3)

$$\omega(q) = cq = \hbar q^2 / 2m, \qquad q^* = 2mc/\hbar.$$

In addition to one phonon interactions there are multiple phonon interactions occurring at higher input energies that can also produce UCN. These are the main subject of this work.

UCN production in Helium can be calculated as follows: the differential cross section for neutron scattering is given by the Fourier transform of the van Hove correlation function $S(q, \omega)$ [16]:

$$\frac{d\sigma}{d\omega} = b^2 \frac{k_2}{k_1} S(q, \omega) \, d\Omega \tag{4}$$

and $S(q, \omega)$ has been measured in great detail [9]

$$d\Omega = 2\pi \sin\theta \, d\theta = 2\pi \frac{q \, dq}{k_1 k_2}. \tag{5}$$

Then

$$\frac{d\sigma}{d\omega} = 2\pi b^2 \frac{k_2}{k_1} S(q,\omega) \frac{q\,dq}{k_1 k_2} = 2\pi b^2 S(q,\omega) \frac{q\,dq}{k_1^2}. \tag{6}$$

Since the limits on q are:

$$k_1 - k_2 < q < k_1 + k_2,$$
$$k_2 = k_u \ll k_1, \qquad q \sim k_1 \quad (\hbar k_u \text{ is the UCN momentum})$$

we have $dq = 2k_u$. Then

$$\frac{d\sigma}{d\omega} = 4\pi b^2 \frac{k_u}{k_1} S\left(k_1, \omega = \frac{\alpha k_1^2}{2}\right), \tag{7}$$

where we assumed $S(q,\omega)$ is constant over the narrow range dq:

$$\omega = \frac{\hbar(k_1^2 - k_2^2)}{2m} = \frac{\alpha}{2}\left(k_1^2 - k_2^2\right) \approx \frac{\alpha}{2} k_1^2.$$

The UCN production rate is given by

$$P(E_u)\,dE_u = \left[\int \frac{d\Phi(E_1)}{dE} N_{\text{He}} \frac{d\sigma}{d\omega}(E_1 \to E_u)\,dE_1\right] dE_u \tag{8}$$

then

$$\int_0^{E_c} P(E_u)\,dE_u = N_{\text{He}} 4\pi b^2 \alpha^2 \left[\int \frac{d\Phi(k_1)}{dE} S\left(k_1, \omega = \frac{\alpha k_1^2}{2}\right) dk_1\right] \int_0^{k_c} k_u^2\,dk_u$$

$$= N_{\text{He}} 4\pi b^2 \alpha^2 \left[\int \frac{d\Phi(k_1)}{dE} S\left(k_1, \omega = \frac{\alpha k_1^2}{2}\right) dk_1\right] \frac{k_c^3}{3} \text{ UCN cm}^{-3}\,\text{s}^{-1}, \tag{9}$$

where E_c, k_c are the critical UCN energy and wave vector of the walls of the storage chamber and $\frac{d\Phi(k_1)}{dE} = \frac{d\Phi(E_1 = \alpha k_1^2/2)}{dE}$ is the energy spectrum of the incoming flux.

Note $S(k_1, \omega)$ is in $(\text{meV})^{-1}$.

2.2. Single phonon production of UCN

The one phonon production rate [1,2,12] is found by evaluating the integral in Eq. (9) over the one phonon peak. (Note that $k^* = 0.7$ Å^{-1}.)

$$P_{1p} = N 4\pi b^2 S^* \alpha \beta \frac{k_c^3}{3k^*} \frac{d\Phi(E_1^*)}{dE} \text{ UCN cm}^{-3}\,\text{s}^{-1}, \tag{10}$$

where $S^* = S_1(k^*) = 0.1$ [9,15] and $\beta = \frac{v_n^*}{v_n^* - c_g^*} = 1.45$ [1,12,13] with v_n^* the neutron velocity at the critical energy for UCN production, E^*, and c_g^* the phonon group velocity at the critical energy. We have replaced $\int_P S(q, \omega = \frac{\alpha}{2}q^2)\,d\omega$ where the subscript means the integration is carried out across the phonon peak with $\beta S_1(k^*)$ where
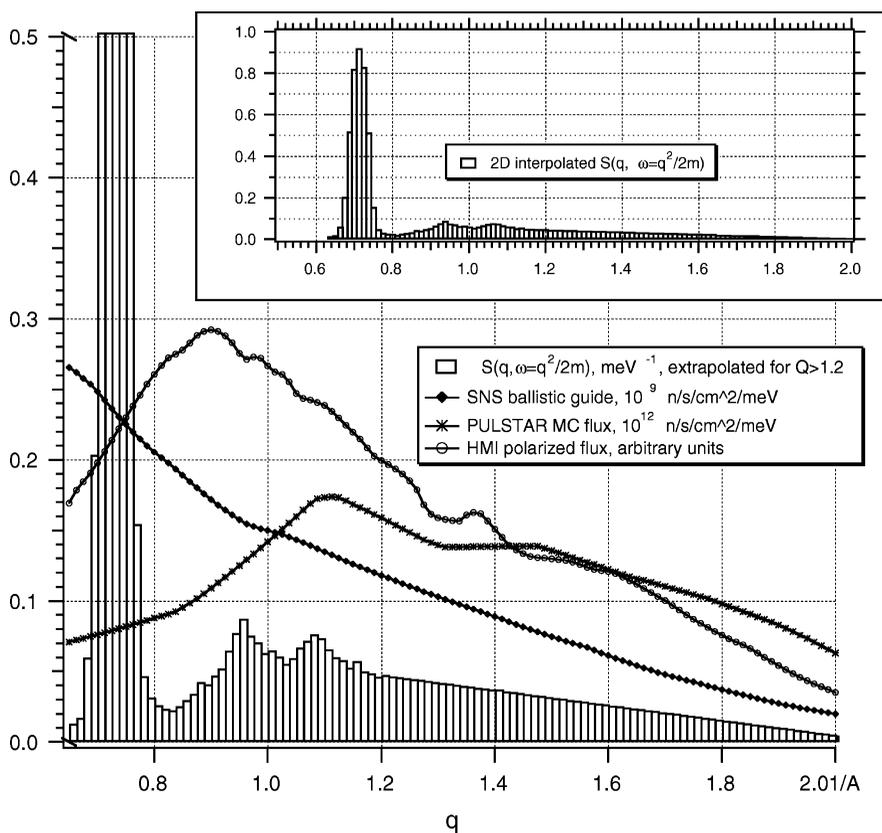
$$S_1(k^*) = \int_P S(k^*, \omega)\,d\omega \tag{11}$$

Fig. 1. The energy spectrum of the neutron flux from three sources compared to the scattering law, $S(q, \omega = \frac{\alpha q^2}{2})$ meV^{-1} as a function of q, Å^{-1}.

is the contribution to the structure factor from the one phonon scattering [9,15] and β represents the increase in the integral due to the fact that the two curves (2) and (3) cross at an angle, so the path of integration is longer than in the calculation of (11), as was first pointed out by Pendlebury [13]. Numerical integration of the measured $S(q, \omega = \frac{\alpha}{2} q^2)$ [9] across the phonon peak (Fig. 1) confirms this procedure. (Note that the factor of 2 in Eq. (9) of [2], which holds in the case of a linear dispersion relation should be replaced by $\beta = 1.45$ for the real, non-linear dispersion relation.)

Thus (see Eq. (10))

$$P_{\text{UCN}} = 9.44 \times 10^{-9} \frac{d\Phi(E_1^*)}{dE_1^*} \text{ UCN cm}^{-3} \text{ s}^{-1}, \tag{12}$$

where $\frac{d\Phi(E_1^*)}{dE_1^*}$ is in units of $n \text{ cm}^{-2} \text{ s}^{-1} \text{ meV}^{-1}$.

2.3. Numerical calculations of the multiphonon down scattering cross section

Fig. 1 shows $S(q, \omega = \frac{\alpha q^2}{2})$ as a function of q obtained from [9]. We have extrapolated the data above 1.2 Å^{-1} in Fig. 1, taking into account the known value of $S(q)$ for the one phonon and multiphonon contributions [9,15]. Measurements [10] show that $S(k_1, \omega = \frac{\alpha k_1^2}{2})$ is essentially zero for $k_1 \gtrsim 2$ Å^{-1}.

As an illustration we apply the above technique to various measured and calculated slow neutron spectra that are under discussion as locations for various superthermal UCN source projects. We will take $k_c = 1.1 \times 10^{-2}$ Å^{-1} corresponding to the critical energy for beryllium.

2.3.1. North Carolina State proposed UCN source

North Carolina State University researchers have proposed to locate a UCN source in the thermal column of the campus 1-MW PULSTAR reactor after removing the graphite. The expected average cold neutron flux in the UCN converter was estimated using MCNP simulations with a detailed model of the PULSTAR core and a conceptual model of the UCN source. The core's 4 control blades and 25 fuel assemblies, each consisting of a 5×5 array of fuel pins containing 4% enriched UO2, were faithfully represented. The model of the UCN source consisted of a cold neutron source surrounding a UCN converter about 10-cm deep in a tank of D2O in the thermal column void. The cold source moderator was 1-cm-thick cup-shaped solid methane (408 g, 22 K) contained in a 2-mm-thick-wall aluminum chamber (1164 g). The UCN converter was modeled by a disk of liquid ortho-deuterium (4-cm thick $\times$ 14-cm diameter, 20 K) in a 5-mm-thick-wall aluminum container (484 g). Neutrons leaving a bare face of the reactor core were channeled into the D$_2$O tank by a 44-cm diameter $\times$ 75-cm long void in a beryllium assembly located between the reactor core and the reactor tank wall. The simulated spectrum is shown in Fig. 1, along with the spectrum of some other sources discussed below and $S(q, \omega = \frac{\alpha q^2}{2})$.

For this spectrum

$$\left[\int \Phi(E_1) S\left(k_1, \omega = \frac{\alpha k_1^2}{2} \right) dk_1 \right] = 5.22 \times 10^9.$$

Substituting into Eq. (9):

$$N = 2.18 \times 10^{22}$$

we have

$$P_{mp} = N \times 4\pi b^2 \times 5.22 \times 10^9 \times \alpha^2 \frac{k_c^3}{3} = 984 \text{ UCN cm}^{-3}\,\text{s}^{-1}.$$

For the one phonon contribution we have, taking $\frac{d\phi}{dE_{\mathrm{meV}}}|_{E^*}$ from Fig. 1 and using Eq. (12)

$$P_{1p} = 9.44 \times 10^{-9} \left. \frac{d\phi}{dE_{\mathrm{meV}}} \right|_{E^*} = 9.44 \times 10^{-9} \times 7.9 \times 10^{10} = 745 \text{ UCN cm}^{-3}\,\text{s}^{-1}.$$

These figures will be reduced by a factor of 2 when the flux (calculated here for a modest source volume as described above) is averaged over a more reasonable 20 liter volume. The effects of construction materials in a realistic design have not yet been taken into account.

The advantages of such a design were discussed in [17]. The main point is that the UCN production process is independent of the direction of the incident neutrons so that use is made of the full solid angle of the flux. Installations at a beam position, as discussed in the next section, lose a solid angle factor of about 10^4 [2] in comparison to installations inside a 4π flux, although such sources outperform any other type of UCN source that could be installed at the same beam position.

2.3.2. Proposed UCN source at the spallation neutron source (SNS)

It has been proposed to place a superthermal source of UCN on a monochromatic 8.9 Å beam at a guide tube at the SNS [6]. It is interesting to ask what the loss of UCN production rate associated with the use of a monochromatic beam would be in the case of the simulated spectra available for the planned guide tubes. Spectra have been simulated for two types of guide: an 'ordinary' supermirror guide and a so-called 'ballistic' guide using tapered sections to focus and defocus the beam at the borders of a long free-flight region. This is an interesting
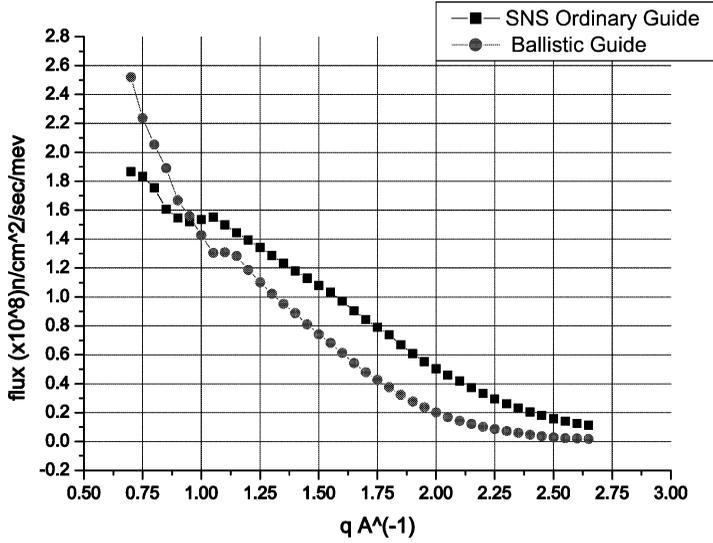
Fig. 2. Comparison of simulated spectra from the two types of guide proposed for the SNS fundamental physics position as a function of q, Å^{-1}.

case for the present discussion as the 'ballistic' guide has more flux at the critical 8.9 Å wavelength but less flux at shorter wavelengths ($\lambda < 6$ Å). The predicted fluxes for the two types of guide are shown in Fig. 2, while Fig. 1 shows a comparison of the SNS ballistic guide with the other sources considered in this Letter.

The result for the 'ordinary' guide is that

$$\int \Phi(E_1) S\left(k_1, \omega = \frac{\alpha k_1^2}{2}\right) dk_1 = 5.38 \times 10^6$$

so that

$$P_{mp} = N \times 4\pi b^2 \times 5.38 \times 10^6 \times \alpha^2 \frac{k_c^3}{3} = 1.015 \text{ UCN cm}^{-3} \text{s}^{-1}.$$

For the one phonon production rate we find using (12)

$$P_{1p} = 9.44 \times 10^{-9} \left.\frac{d\phi}{dE_{\text{meV}}}\right|_{E^*} = 1.8 \text{ UCN cm}^{-3} \text{s}^{-1}.$$

For the 'ballistic' guide we have

$$\int \Phi(E_1) S\left(k_1, \omega = \frac{\alpha k_1^2}{2}\right) dk_1 = 5 \times 10^6$$

so that $P_{mp} = 0.94$ and the one phonon production rate is: $2.36 \text{ UCN cm}^{-3} \text{s}^{-1}$.

Thus we see that the losses associated with rejecting the higher energy neutrons in the beam by the monochromator are less significant than the losses (50%) [6] connected with the monochromator and associated beam bender.

2.3.3. HMI beam spectrum

This spectrum was measured at the NL4 neutron guide at the Hahn Meitner Institut, Berlin by time of flight. In this context we should remind the reader that the measurement of the spectrum from a neutron guide is quite complex as the spectra in general are very strong functions of the flight direction and position in the beam cross
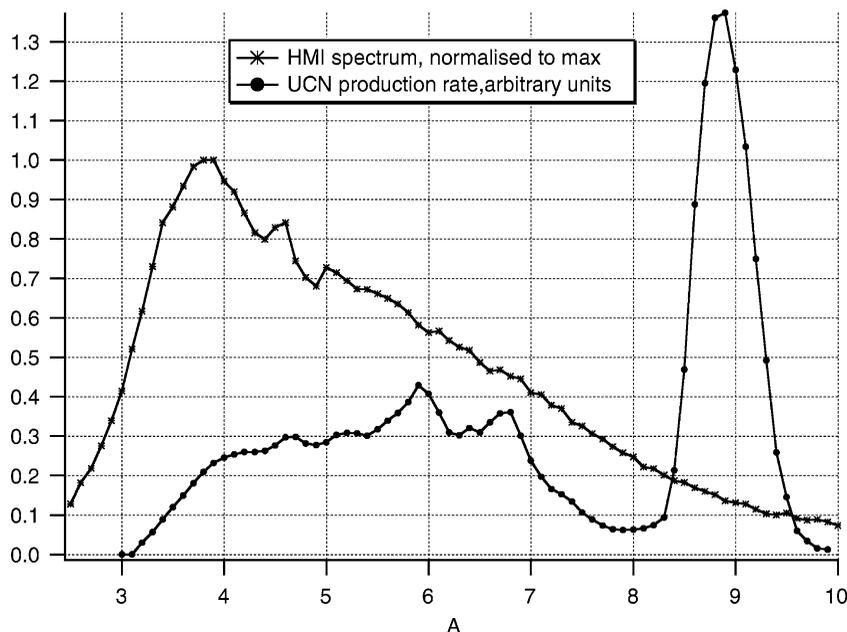
Fig. 3. The energy spectrum of the flux measured on the beam NL4 of the HMI and the calculated UCN production as a function of λ–Å.

section. For the measured spectrum (normalized to 1 at the peak of $d\phi/d\lambda$) we find:

$$\left[\int \Phi(E_1)S\left(k_1, \omega = \frac{\alpha k_1^2}{2}\right)dk_1\right] = 0.025$$

yielding

$$P_{mp} = 4.72 \times 10^{-9}$$

the one phonon contribution for this spectrum is found Eq. (12) as

$$P_{1p} = 5.48 \times 10^{-9}.$$

Fig. 3 shows the spectrum and calculated UCN production rate as a function of the wavelength of the incident neutrons, λ, similar to what would be observed in a time of flight measurement.

3. Discussion

The results are summarized in the table below. The column labelled 'Maxwell' refers to a Maxwellian spectrum corrected for an ideal guide transmission $\tau \propto \lambda^2$ and cut off at 3.8 Å minimum wavelength.

We see that the multiphonon contribution as expected is a rather strong function of the source spectrum varying from less than to slightly more than the one phonon production rate for the realistic spectra considered here (Table 1).

(The NC State figures have been reduced by the factor of 2 for a 20 liter source volume as discussed above.)

Thus we see that the inclusion of the multiphonon production amounts to at most a little more than a factor of 2 increase in UCN production. In the case of a cold beam the multiphonon contribution is a small correction to the single phonon production so that the use of a monochromatic beam, which offers significant operating advantages [6], would be accompanied by a minor loss in UCN production.

Table 1
Predicted production rates

	NC State [a]	SNS ord [a]	SNS ball [a]	HMI a.u.	Maxwell
Multi-ph	490	1.0	0.94	4.7	1.7
Single-ph	375	1.8	2.4	5.5	1.5
Mph/1ph	1.4	0.55	0.4	0.85	1.13

[a] $UCN\,cm^{-3}\,s^{-1}$.

In contrast, for sources where the Helium is exposed to the total thermal flux as in the North Carolina State proposal or at a dedicated spallation source, the multiphonon contribution can amount to slightly more than a factor of 2 increase in UCN production.

References

[1] R. Golub, D.J. Richarson, S.K. Lamoreaux, Ultra-Cold Neutrons, Hilger, 1991;
    V.K. Ignatovich, The Physics of Ultracold Neutrons, Clarendon, Oxford, 1990.
[2] R. Golub, J.M. Pendlebury, Phys. Lett. 82A (1977) 337.
[3] P.R. Huffman, et al., Nature 403 (2000) 62.
[4] R. Golub, S.K. Lamoreaux, Phys. Rep. 237 (1994) 1.
[5] P. Barnes, in: A New Search for the Electric Dipole Moment of the Neutron, Los Alamos National Laboratory, 2002.
[6] Fundamental Phys. Instrument Team and Phys. Division Oak Ridge National Laboaratory, Proposal for Fundamental Neutron Phys. Beamline at the Spallation Neutron Source (SNS), 2002.
[7] S.K. Lamoreaux, R. Golub, JETP Lett. 58 (1993) 792.
[8] H. Yoshiki, et al., Phys. Rev. Lett. 68 (1992) 1323.
[9] M.R. Gibbs, et al., J. Low Temp. Phys. 120 (2000) 55;
    M.R. Gibbs, et al., J. Phys. Condens. Matter. 11 (1999) 603;
    K.H. Andersen, et al., J. Phys. Condens. Matter. 6 (1994) 821;
    K.H. Andersen, private communication, 2001.
[10] B. Fåk, K.H. Andersen, Phys. Lett. A 160 (1991) 468.
[11] R. Golub, et al., Z. Phys. B 51 (1983) 187;
    A.I. Kilvington, et al., Phys. Lett. A 125 (1987) 416.
[12] C.I. Jewell, The interaction of UCN with superfluid He^4 and a new possible super thermal UCN source, Thesis, University of Lancaster, 1983.
[13] J.M. Pendlebury, private communication, 1982 (unpublished manuscript).
[14] Y. Abe, N. Morishima, Nucl. Instrum. Methods Phys. Res. A 463 (2001) 293.
[15] R.A. Cowley, A.D.B. Woods, Canadian J. Phys. 49 (1971) 177.
[16] W.C. Marshall, S.W. Lovesey, Theory of Thermal Neutron Scattering, Clarendon, 1971.
[17] R. Golub, Nucl. Instrum. Methods 226 (1984) 558.
[18] Y. Masuda, Nucl. Instrum. Methods Phys. Res. A 440 (2000) 682;
    Y. Masuda, et al., He-II Spallation UCN source, in: Proc. ICNANS-XV, KEK Proceedings 2000-22 (2001) 1067.