Ultracold neutron depolarization in magnetic storage traps

Santa Fe, Nov. 9, 2012

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For magnetic UCN storage used to measure the neutron lifetime

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Theory: Majorana 1932; Walstrom *et al*., 2009

Experimental: Paul *et al*., 1989

Huffman *et al*., 2000

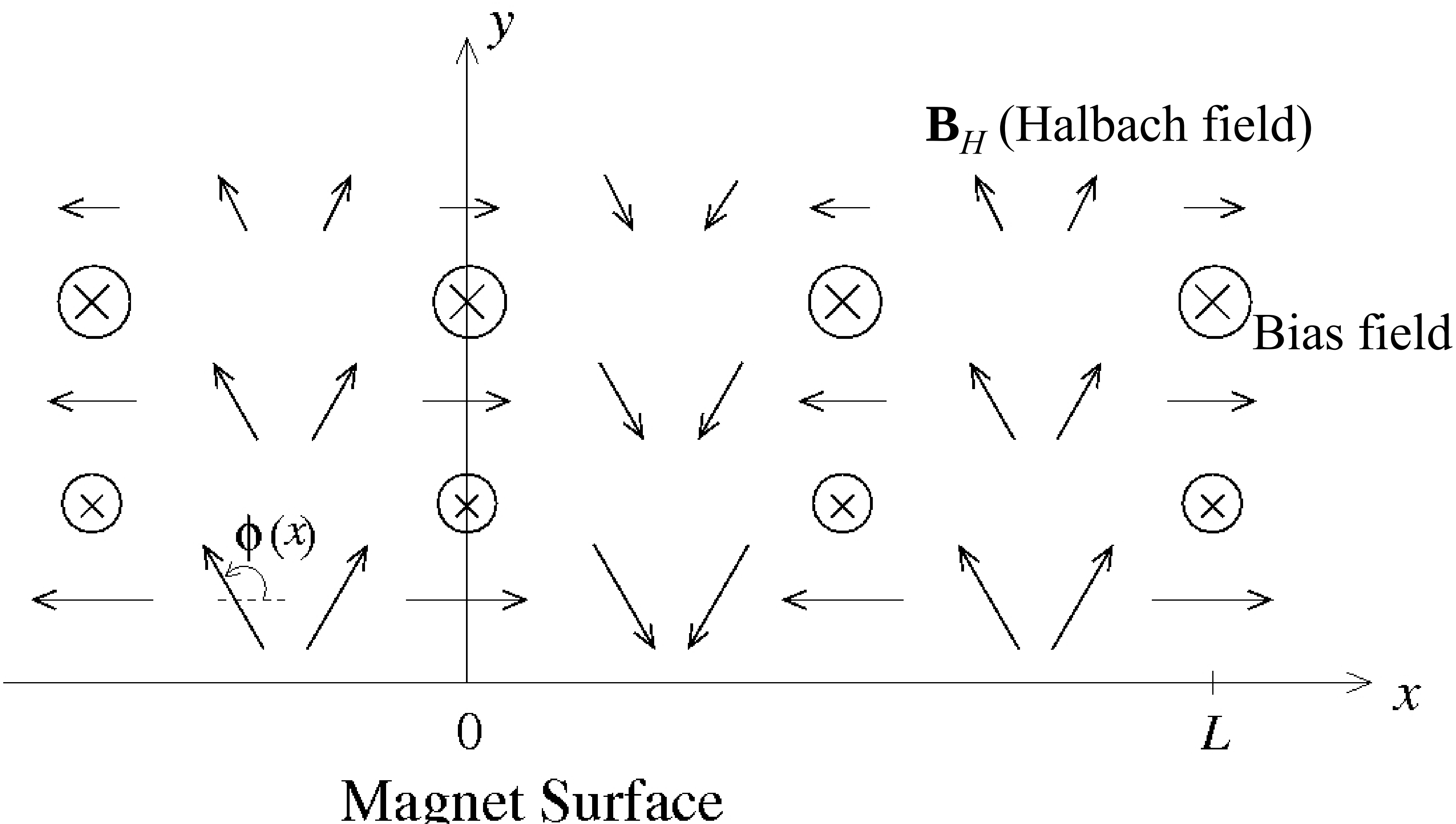
Leung, Zimmer, 2009 HOPE

Walstrom *et al*., 2009 "bathtub"

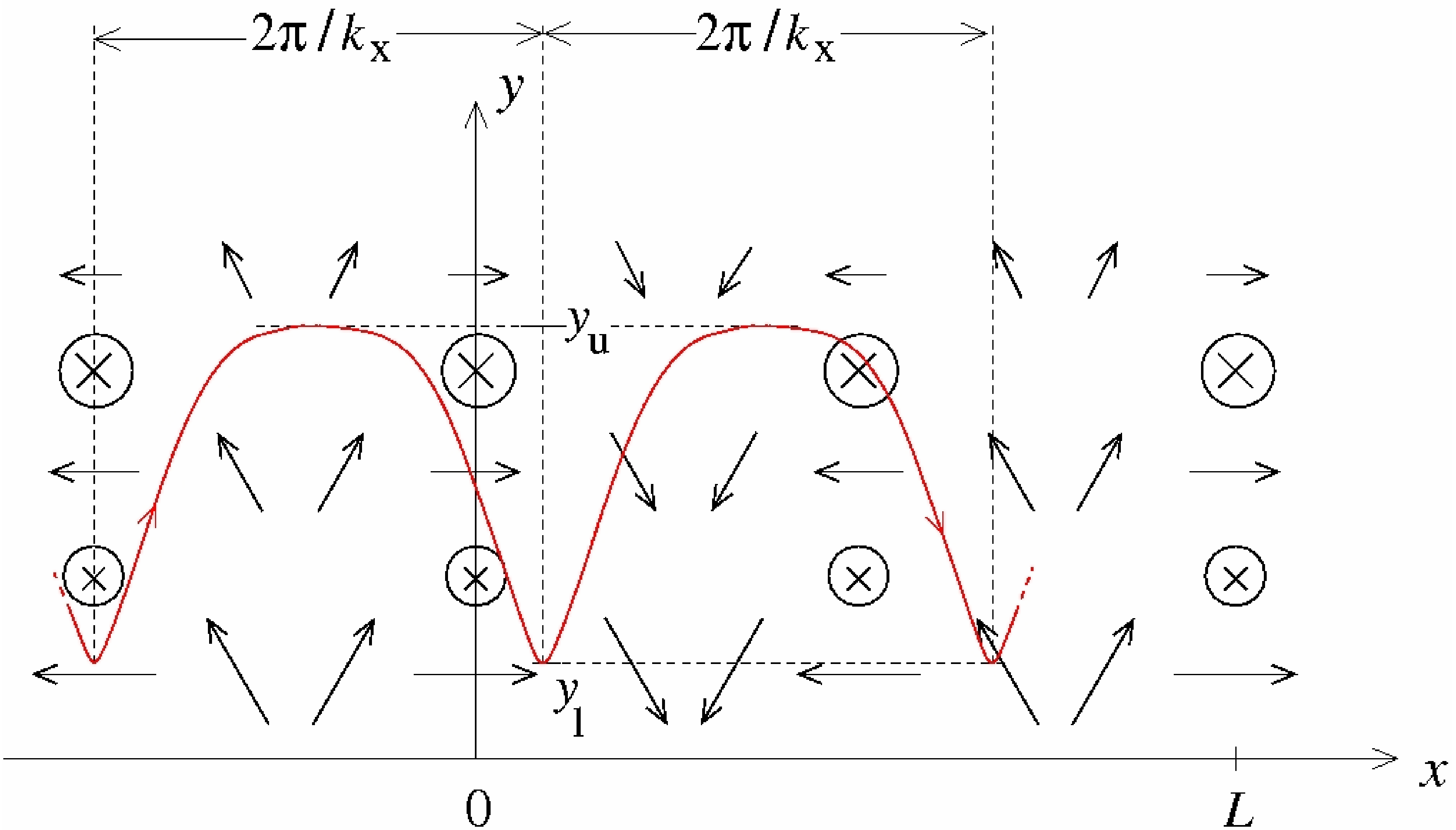
Materne *et al*., 2009 PENELOPE

J-Parc

Our magnetic field model (based on Los Alamos "bathtub" project, but 1D)



UCN path in 1D magnetic field model



Magnet Surface

Interpretation is critical

UCN wave function χ is a superposition

$$\chi = \alpha\,|+\rangle + \beta|-\rangle$$

$|+\rangle$ is storable; $|-\rangle$ tends to escape

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UCN wave function χ is a superposition

$$\chi = \alpha\,|+\rangle + \beta|-\rangle$$

$|+\rangle$ is storable; $|-\rangle$ tends to escape

As the UCN moves around, $|\beta|^2$ changes periodically by many orders of magnitude.

What is the measurable depolarization (per second or per bounce in the field)?

For initial $|+\rangle$ state, what is the measurable depolarization D?

Majorana: D = value of $|\beta|^2$ at end of classical path from $t = -\infty$ to $t = +\infty$

Walstrom *et al.*: D = value of $|\beta|^2$ at classical turning points of vertical up or down paths (for special case $v_z = v_x = 0$)

Our extension to arbitrary v_z, v_x: D = value of outgoing current $\mathrm{Re}(\beta^* \nabla_y \beta / i)$ at classical turning surfaces for given energy of vertical motion

Analysis of depolarization in non-uniform **B**-field

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(a) extension of semi-classical approach

(b) quantum analysis of space and spin dependence using the WKB approximation

Analysis of depolarization in non-uniform **B**-field and in mirror reflection

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Our approach to arbitrary 3D motion ($v_x, v_z \neq 0$):
- (a) extension of quantum analysis of evolution in 3D space and spin space using the WKB approximation
- (b) further extension of semi-classical approach to 3D
- (c) place a non-magnetic mirror into non-uniform magnetic field; determine probability of UCN depolarization per reflection using methods (a) and (b)

A glimpse of the math

Solve the Schrödinger equation for the superposition wave

$$\chi = \alpha^{(3)}\chi^{+} + \beta^{(3)}\chi^{-}$$

$$\chi^{-} = \begin{pmatrix} e^{-i\phi}\sin\frac{\theta}{2} \\ -\cos\frac{\theta}{2} \end{pmatrix};\ \chi^{+} = \begin{pmatrix} \cos\frac{\theta}{2} \\ e^{i\phi}\sin\frac{\theta}{2} \end{pmatrix}$$

χ^{+}, χ^{-}: basis vectors for spin parallel/antiparallel to field

θ = polar angle between **B** and the *z*-axis

ϕ = azimuthal angle for Halbach field (in the *xy*-plane)

QMech: $\alpha^{(3)} = \alpha^{(3)}(x,y,z) = \alpha(y)\ \exp(ik_x x)\ \exp(ik_z z)$

$\beta^{(3)} = \beta^{(3)}(x,y,z) = \beta(y)\ \exp(-iKx)\ \exp(ik_x x)\ \exp(ik_z z)$

CLass: $\alpha^{(3)} = \alpha(t)$; $\beta^{(3)} = \beta(t)$;

A glimpse of the math

For QU: Use the WKB approximation for α and β and solve the inhomogeneous diff. equation

$$\beta''(y) + k_-^2(y)\beta(y) = \theta'(y)\alpha'(y) + Kk_x\alpha(y)\sin\theta(y)$$

new, dominant term

WKB solution: $$\beta'' = -k_+^2\beta \Rightarrow \beta \approx \frac{\theta'\alpha' + Kk_x\alpha\sin\theta}{k_-^2 - k_+^2}$$

satisfies the initial condition at turning point y_s (start from pure + spin state)

$(\ )' = d(\ )/dy$; $K = 2\pi/L$; $\mathbf{k} = m\mathbf{v}/\hbar$;

$k_\pm$ = vertical component of **k** for ± spin state;

$$\alpha(y) = \frac{1}{\sqrt{k_+(y)}}\exp\left(\pm i\int_{y_s}^{y} k_+(y')dy'\right) \rightarrow \alpha'(y) \cong \pm i\sqrt{k_+(y)}\exp\left(\pm i\int_{y_s}^{y} k_+(y')dy'\right)$$

A glimpse of the math

For CL: Solve the inhomogeneous diff. equation

$$\dot{\beta}(t) - \frac{i\omega_L}{2}\beta(t) = e^{-iKv_x t}\frac{\alpha(t)}{2}[\dot{\theta}(t) - iKv_x \sin\theta(t)]$$

new   new, dominant term

Approx. solution:

$$\dot{\beta} = -\frac{i\omega_L}{2}\beta \Rightarrow \beta \approx \frac{i}{2\omega_L}e^{-iKv_x t}\alpha(t)(\dot{\theta} - iKv_x \sin\theta)$$

$\omega_L = 2|\mu_n|B/\hbar$ = Larmor frequency

$\dot{(\cdots)} = d(\cdots)/dt$

$$\alpha(t) = \exp\left(-\frac{i}{2}\int_{t_s}^{t}\omega_L(t')dt'\right)$$

Results: (a) depolarization in non-uniform magnetic field

For given v_x, v_z and fixed energy for vertical motion

QU current
= depol. probability: $\frac{m}{\hbar} j = \mathrm{Re}[\beta^* \frac{1}{i} \nabla \beta] = \frac{k_+^2 \theta'^2 + K^2 k_x^2 \sin^2\theta}{(k_-^2 - k_+^2)^2}$

CL: depol. probability: $|\beta|^2 = \frac{k_+^2 \theta'^2 + K^2 k_x^2 \sin^2\theta}{(k_-^2 - k_+^2)^2}$

$$k_-^2 - k_+^2 = \frac{2m}{\hbar} \omega_L$$

Dependence of depolarization on position y between y_u and y_l

Walstrom *et al.* (2009)

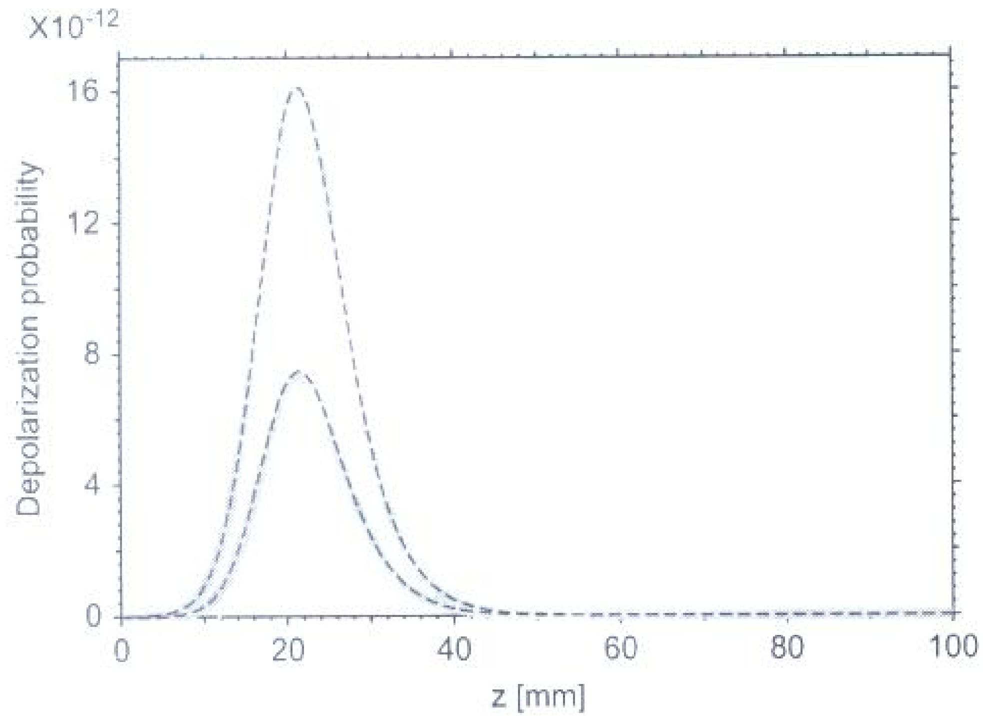
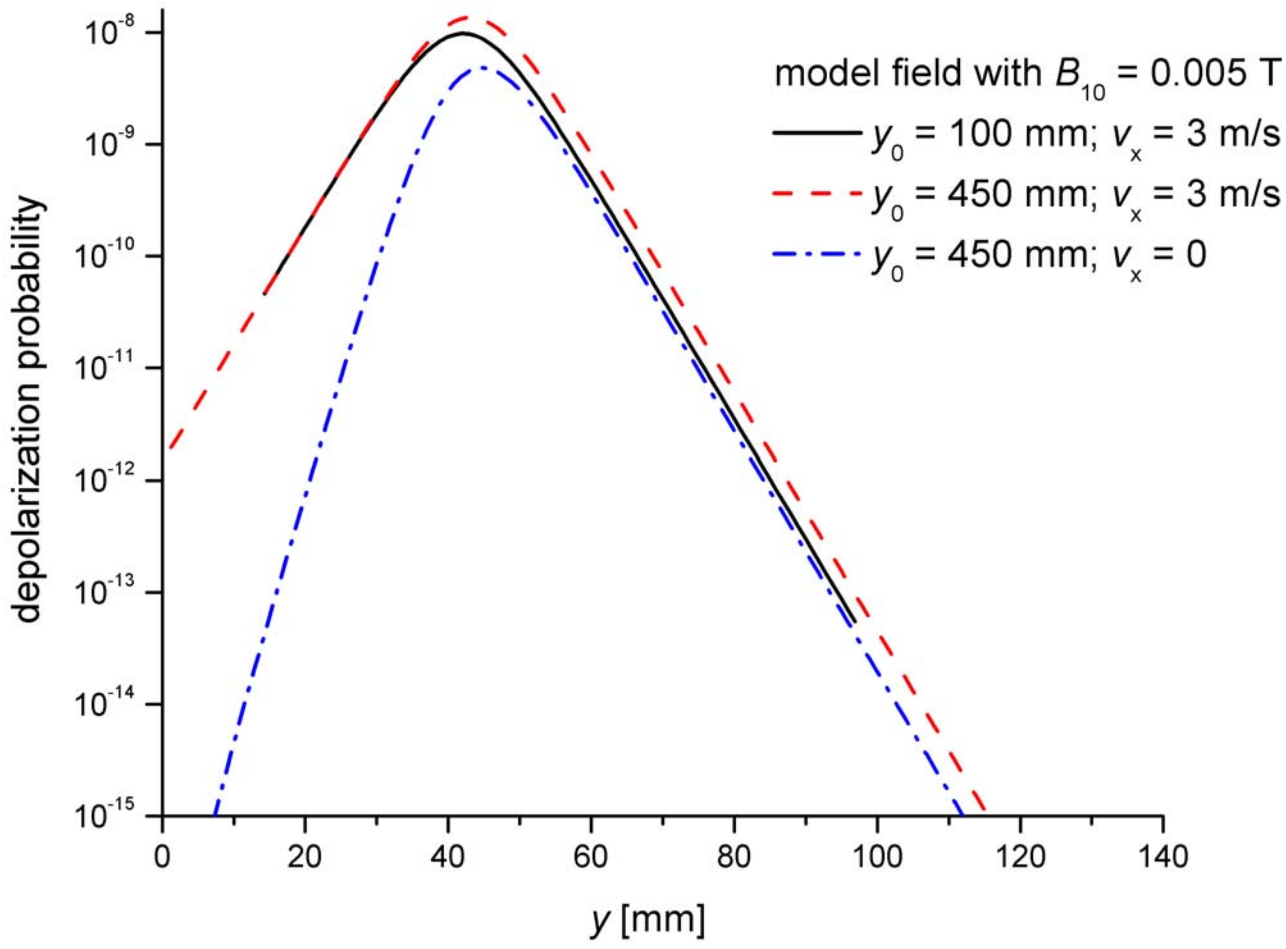


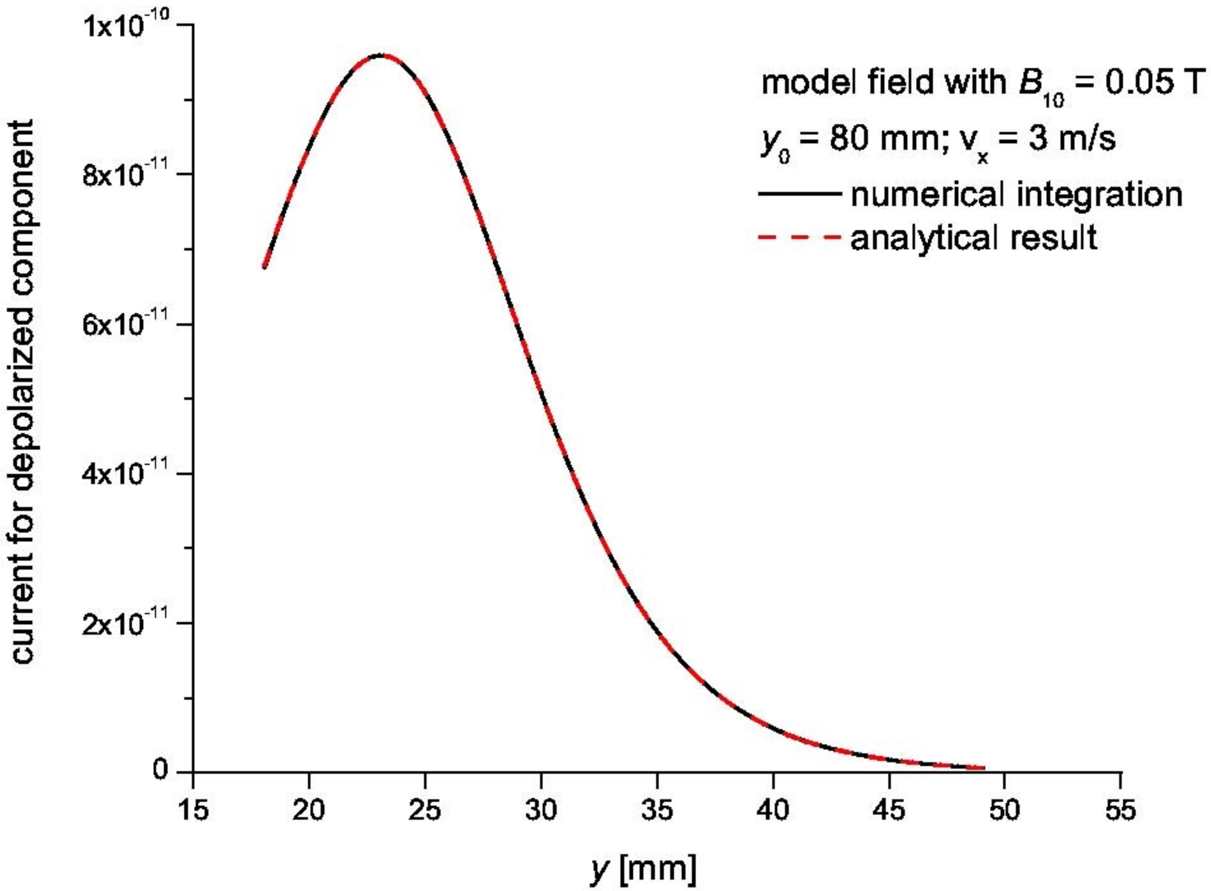
Fig. 3. Squared depolarization amplitude for two drop heights in the semiclassical (gray curves) and WKB (black dashed curves) approximations. The upper and lower curves were computed for a drop heights of 0.5 and 0.25 m, respectively.

Dependence of depolarization on position *y* between y_u and y_l



Dependence of depolarization on position y between y_u and y_l

Numerical integration vs. analysis



Results: (a) depolarization in non-uniform magnetic field

For given v_x, v_z and fixed energy for vertical motion

Loss per bounce (for full move down and up):

both for QU and CL:

$$\frac{m}{\hbar}\langle j_l + j_u \rangle = K^2 k_x^2 \left(\frac{\sin^2\theta_l}{k_{-l}^4} + \frac{\sin^2\theta_u}{k_{-u}^4}\right) \propto \omega^2 \left(\frac{B_{Hl}^2}{B_l^4} + \frac{B_{Hu}^2}{B_u^4}\right)$$

vanishes for vertical drop $(k_x = 0, \omega = 0)$

Depolarization loss (per s):

both for QU and CL: $\tau_{dep}^{-1} = \dfrac{m}{\hbar}\dfrac{\langle j_l + j_u \rangle}{T}$

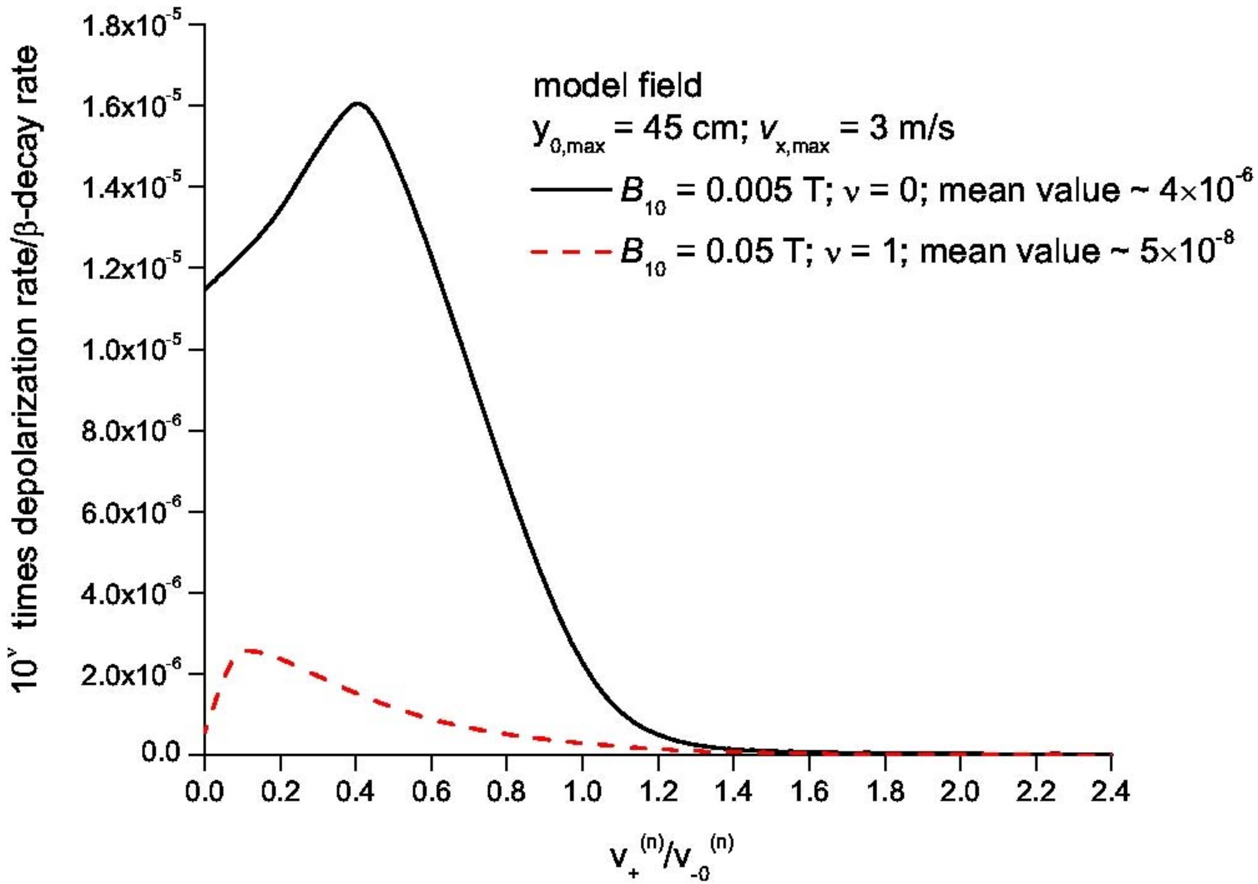
Index l/u designates lower/upper turning point;

$\omega = Kv_x$ = frequency of Halbach field in the moving reference frame;

T = time for one round trip down and up.

Depolarization rate (per s) divided by β-decay rate

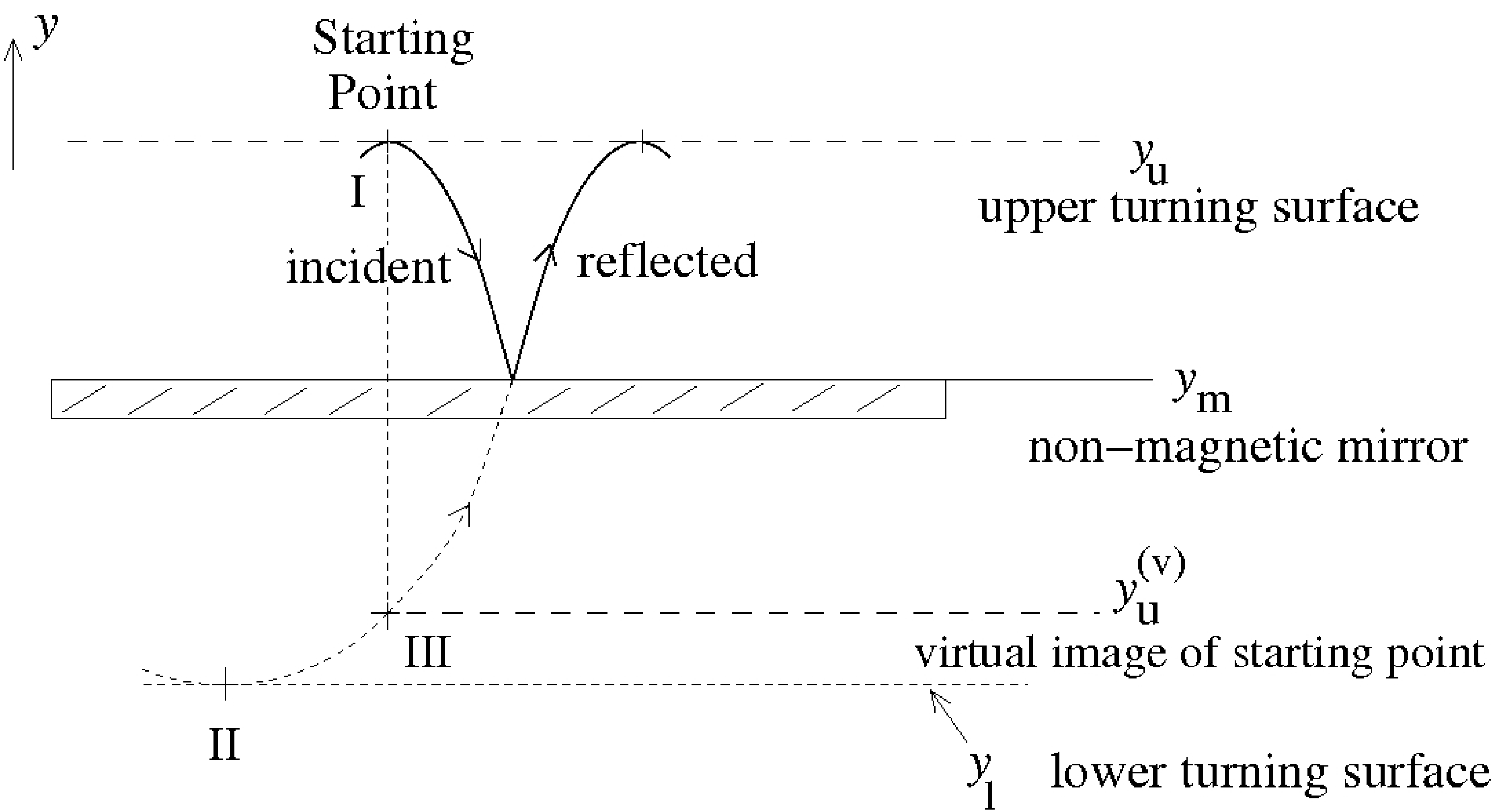
For a Maxwell spectrum: average the curves



4×10^{-6}

5×10^{-8}

(b) Depolarization in reflection from an ideal non-magnetic mirror



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Recall the inhomogeneous diff. equations for β

QU: $$\beta''(y) + k_-^2(y)\beta(y) = \theta'(y)\alpha'(y) + Kk_x\alpha(y)\sin\theta(y)$$

CL: $$\dot{\beta}(t) - \frac{i\omega_L}{2}\beta(t) = \frac{\alpha(t)}{2}[\dot{\theta}(t) - iKv_x\sin\theta(t)]e^{iKv_x t}$$

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Their general solution is: $$\beta = \beta_p + \beta_h$$

β_p: particular solution

β_h: solution of the homogeneous equation

(b) Depolarization in reflection from a low-loss non-magnetic mirror

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Strategy: A particular solution β_p for the reflected wave is known. Adjust amplitude of homogeneous solution β_h to satisfy matching condition at the mirror surface.

Results for depolarization in reflection from a low-loss non-magnetic mirror

CL: $$p = \frac{(Kk_x \sin\theta_u)^2}{k_{-u}^4} + 2\frac{(k_{+m}\theta'_m)^2 + (Kk_x \sin\theta_m)^2}{(k_{-m}^2 - k_{+m}^2)^2}$$

QU: $$p = \frac{(Kk_x \sin\theta_u)^2}{k_{-u}^4} + 2\frac{k_{-m}}{k_{+m}}\frac{(k_{+m}\theta'_m)^2 + (Kk_x \sin\theta_m)^2}{(k_{-m}^2 - k_{+m}^2)^2}$$

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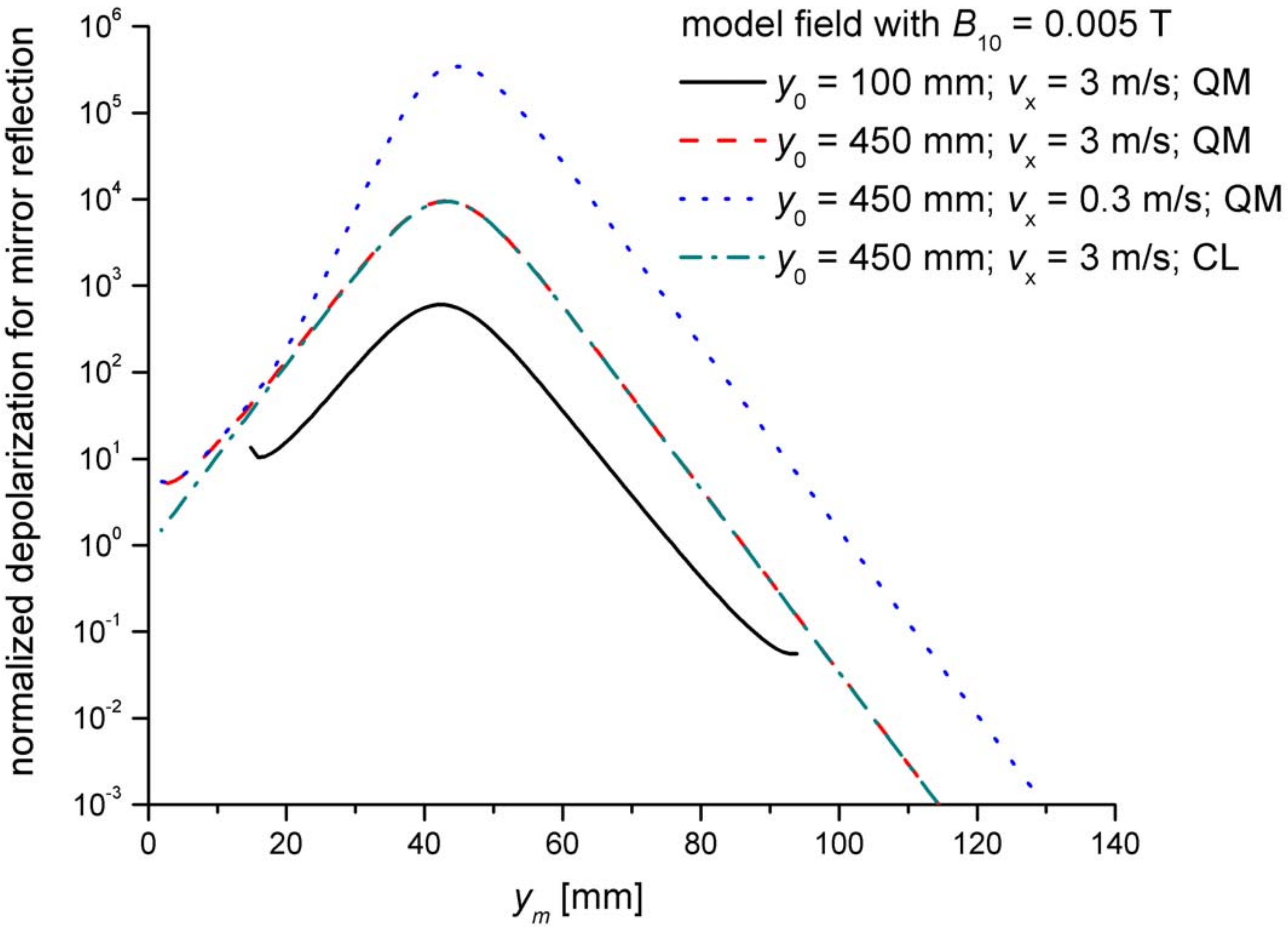

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In typical cases: $k_{-m}/k_{+m} \approx 1$ and the first term for p is negligible

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In typical cases: $k_{-m}/k_{+m} \approx 1$ and the first term for p is negligible.

The second term is proportional to the depolarization current in the field.

⇒A mirror acts like a polarization analyzer (for any mirror orientation and curvature).

Conclusions

(a) UCN depolarization in magnetic trap

Majorana: ~$\exp(-10^6)$ (= depolarization for one passage through the field)

Walstrom *et al.*: ~(10^{-20}-10^{-23}) (= depolarization for one vertical bounce)

Our result ~10^{-8} s^{-1} (= depolarization rate for our 1D model field, with bias field B_{10} = 5 mT and v_x = 3 m/s)

Conclusions

(b) Probability of depolarization in UCN reflection from non-magnetic mirror in a non-uniform magnetic field

Pokotilovsky (2002, matching Majorana solutions before and after reflection):

$p \sim \omega_{Lm}^{-2}$ can be large

ω_{Lm} = Larmor frequency at mirror location

Conclusions

(b) Probability of depolarization in UCN reflection from non-magnetic mirror in a non-uniform magnetic field

Pokotilovsky (2002, matching Majorana solutions before and after reflection):

$p \sim \omega_{Lm}^{-2}$ can be large

Our result: Non-magnetic mirror in a non-uniform **B**-field acts like a polarization analyzer, measuring the wave component for spin flip

$$p = \frac{k_+^2 \theta'^2 + K^2 k_x^2 \sin^2 \theta}{(k_-^2 - k_+^2)^2}$$

p is large for small $B \sim k_-^2 - k_+^2$, large θ′ and/or large $\omega \frac{B_H}{B}$

Conclusions

(b) Probability of depolarization in UCN reflection from non-magnetic mirror in a non-uniform magnetic field

Data on depolarization probabilities for different mirror materials and temperatures show remarkably similar values.

Such independence is expected for our model if the samples were measured in same non-uniform **B**–field.

Can the values ($\sim 10^{-5}$) be explained? Depends on the details of the magnetic field distribution in the sample chamber.